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Science is the labour and handicraft of the mind; poetry can only be considered its recreation.—FRANCIS BACON.

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WE COVER THE EARTH

School Science and Mathematics is read by subscribers in every state of the Union, all provinces of Canada, and thirty-three foreign countries.

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SCHOOL SCIENCE AND MATHEMATICS

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PLAN NOW TO ATTEND THE 1946 CONVENTION

Book-Cadillac Hotel, Detroit

General and Section Meetings, November 29
General and Group Meetings, November 30

Greetings to teachers of science and mathematics!

Each year you examine enthusiastically the annual convention program of the CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS. Each year many of you have the pleasure of attending this convention! Such attendance has paid rich dividends in educational experiences. Again your attention is called to the fact that the 1946 convention is in the offing and preliminary plans indicate a program will be presented which will be timely and inspirational.

The committee in charge is directing its efforts towards building a convention program around the slogan—"New Power, Products, and Personnel." Through talented speakers, exhibits, and demonstrations such concepts as are outlined in the following paragraphs will be translated into practical procedures for teaching science and mathematics:

NEW POWER

Concepts of Air Age and Atomic Age education bring us to new frontiers in modern living. Some of these frontiers will be developed immediately; others, ultimately. While atomic energy may not be harnessed immediately as a practical source of power, research in this field indicates tremendous possibilities for the future. Of more immediate and practical concern is the fact that new and greater POWER HAS BEEN UNLEASHED

through explorations of electronic, turbine, rocket, and jet propulsion principles. Applications of these principles and others can bring us to a great period of industrial expansion, and hence, to revolutionized living.

PRODUCTS

New products, new materials, and new services are in the offing for American people. Prefabricated homes; newer types of automobiles; practical models of Piper Cubs, Ercoupes, and helicopters; streamlined trains and busses; extraordinary household appliances such as washing machines which handle the complete process automatically, improved kitchen stoves, deep freeze units, fluorescent lighting fixtures, and possibly electronic heating units may bring revolutionized post-war living.

All of these factors may result in new jobs, new factories, new labor problems and the need for a much greater application of SCIENCE and MATHEMATICS to the development of such social and political measures as will lead to a society geared to the promotion of better living for all people.

PERSONNEL

World War II has focused attention on the importance of science and mathematics in the training of military personnel. Now that the war is over, we must face the fact that well trained scientists and mathematicians will be needed to help preserve national safety and to permit this Nation to cooperate effectively in the maintenance of world peace.

Furthermore, health and safety will depend, in part, upon medical science reducing the number of deaths from contagious and degenerative diseases. Hence, we must not overlook the training of a sufficient number of people to do basic research. Then, too, as scientific and mathematical principles continue to be applied to industrial expansion, the need for adequately trained personnel becomes apparent.

Hence, we must be ready to give a certain body of our students such high school and college training as will enable them to fit into fields of research and into technical positions in industries; we must be prepared also to train a certain other body of students for less technical and less skilled positions; and we must be ready to train all people, skilled and unskilled, to live and work effectively in the world as we will experience it TODAY and TOMORROW.

For all individuals adequate training in SCIENCE and MATHEMATICS becomes an objective of great importance in modern education.

New sources of POWER bring us to new frontiers! These frontiers, if pioneered in the same spirit with which we won the war and if directed to the promotion of better health, more jobs, and a higher standard of living, will increase our security as a Nation and will promote cultural progress. Hence, such applications of SCIENCE and MATHEMATICS for the Nation and for the individual may be very great.

The committee in charge of the 1946 convention cordially invites all teachers of science and mathematics to attend. The standard rates for the Book-Cadillac Hotel in November will prevail. Why not secure your ROOM RESERVATION NOW by writing directly to Mr. C. B. Loftis, Front Office Manager, Book-Cadillac Hotel, Detroit 31, Michigan?

ARTHUR O. BAKER, *President*
Central Association of Science and
Mathematics Teachers

WESTINGHOUSE FELLOWSHIP AWARDS PRODUCE RESULTS

The discovery of photo-fission, a new type of nuclear fission—briefly the "splitting of uranium atoms by high energy gamma rays with a commensurate release of large amounts of energy—was made in 1940 by three fellows, appointed in 1938 and reappointed the following year, the research director said.

Dr. W. E. Shoupp, one of the co-discoverers, is now manager of the Electronics Department of the laboratories and has been active during the war in the development of radar devices and radar countermeasures. Another, Dr. W. E. Stephens, now is assistant professor of physics at the University of Pennsylvania.

Two important developments that are a direct outcome of fellowship projects are the portable mass spectrometer for speedy analysis of gases, designed by Dr. John A. Hipple, now section engineer in the Electro-Physics Department, and the TR Box, a superspeed electronic switch used in radar equipment, one of whose co-inventors was Dr. Sidney Krasik, now Electronics Department section engineer. Dr. Hipple's first work with the laboratories was to build a room-sized prototype of the portable "atom sorter" and Dr. Krasik's project was research on the fundamentals of velocity-modulated electron beams as generators of ultra high frequency radiations. Continuing in research on magnetism, Dr. Sidney Siegel, section engineer of the Magnetics Department, has notably advanced the theories of magnetism and the knowledge of magnetic materials.

Ten other fellowship winners are still at the laboratories, four have joined other industrial concerns, two are assistant professors at universities, two are with government research laboratories, and one is deceased.

THE COMMERCIAL DRIED FRUIT INDUSTRY

ZOE A. THRALLS
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The dried fruit industry is an ancient industry. Until the 19th century, drying fruit was one of the few ways of preserving it for future use. But only certain fruits will dry satisfactorily. Fruits that dry successfully must be rich in sugar and have a low water content. The pulp must be firm. The fruit should be free from disease and insect injury. Formerly the chief fruits which had these characteristics were dates, figs, prunes, currants, and raisin grapes. Rather recently, types of peaches, apricots, and pears have been developed which also have such characteristics.

Fruits rich in sugar and low in moisture content require rather specific natural conditions if they are satisfactory in quality and quantity. The best quality of such fruit is produced in regions with a dry, sunny climate, where man can control the water supply, and where the soil has a high mineral content. The minerals in the soil help to make the fruit pulp firm.

Until about fifty years ago, the Mediterranean countries—Spain, Italy, Greece, Turkey, Yugoslavia, Algeria, Tunisia, and Morocco produced almost all of the dried fruit sold in the world market. But in the last fifty years, the center of the industry gradually shifted from the lands around the Mediterranean Sea to our southwestern states with California as the leader. The total production of dried fruits has increased tremendously. More people are using dried fruits and using them in larger quantities than ever before, even though canning, refrigerating, and freezing methods of preserving foods have developed. Few people realize how the dried fruit industry has grown and how important it is in the United States and in world trade.

This unit—the *Commercial Dried Fruit Industry*—is for the purpose of developing the following understandings, abilities, and appreciations.

1. An understanding of:

(a) How the United States has attained world leadership in the production of dried fruits;
(b) How this position has been the result of taking advantage of natural conditions favorable to the industry;
(c) How modern scientific methods of cultivation, preparation, and market have aided the industry.

(d) How other regions rank in the dried fruit industry and why they have developed the industry.

2. Ability to locate:

(a) The leading dried fruit producing areas in the United States;

(b) The leading dried fruit producing areas in other parts of the world.

3. Ability to get descriptive information from the following types of maps:

(a) Seasonal rainfall maps;

(b) Temperature and length of growing season maps;

(c) Physical maps;

(d) World trade and trade route maps.

4. Ability to read facts of geographical significance from statistical tables containing pertinent information on the industry.

5. An appreciation of the importance of the dried fruit industry to the United States and in world trade.

6. An appreciation of the value of dried fruits in improving man's diet.

7. An appreciation of the significance of climate and other natural factors in the development of the dried fruit industry.

8. An appreciation of the part played by technological progress in the growth of the industry.

The Commercial Dried Fruit Industry

I. Introduction: How has man preserved fruits for future use?
   - A. In ancient times, how did man preserve fruits for future use?
   - B. What kinds of fruit are suitable for drying? Why?
   - C. What are some modern methods of preserving fruits?
   - D. Suggest some possible reasons why dried fruits remain popular.

II. Why has the dried fruit industry been important in the Mediterranean Region of the Old World for several thousand years?
   - A. What fruits suitable for drying are raised in the lands around the Mediterranean Sea?
   - B. What natural conditions in the Mediterranean lands favor the production and drying of these fruits?

To the Teacher: Have the class use seasonal rainfall, temperature, and physical maps, and also climatic data to get descriptive information on this question. Have them check the

inferences drawn from the maps with the textual material.

III. Why do the lands around the Mediterranean Sea retain world leadership in the production of the currant, the date, the fig?
   A. Why is the date a characteristic product from the oases in the deserts adjoining the Mediterranean Lands?
      1. What natural conditions are necessary for the date tree?
      2. How are date orchards cared for?
      3. How are dates gathered, prepared for market, and marketed?
      4. What are the leading commercial producing oases?
   B. Why is Asia Minor the leading area in the production of dried figs?
      1. What valleys are important?
      2. What natural conditions favor the raising of figs?
      3. How are they raised and prepared for market?
      4. What is the leading marketing center?
   C. Where is the currant produced? Why?

IV. How has California become the world center of the dried fruit industry?
   A. In what respects does California resemble the Mediterranean Lands of the Old World?

To get information on this question, compare the physical map, seasonal rainfall maps, temperature maps and climatic data for California and for the lands around the Mediterranean Sea. Check the descriptive information secured from the maps with information from references.

   B. Why is the apricot one of the more valuable dried fruits produced in California?
      1. Why is the apricot, next to the date, the most expensive dried fruit?
      2. Why is California the only place in the United States where the apricot can be grown commercially?
      3. Where is the chief apricot producing section in California?
   C. Why is California the only center in the world where dried peaches and pears are produced in any quantity?
      1. Where is the chief center in California for the production of dried peaches? Why?
      2. In what California valley are pears raised?

3. Why are pears difficult to grow and to dry properly?

D. Why is the Santa Clara valley the leading prune center?
   1. What are prunes?
   2. What natural conditions are favorable to the production of high quality dried prunes?
   3. Describe the seasonal work on a prune ranch.
   4. How does the production of prunes compare with the production of other dried fruits?

E. Why is Fresno, California, the "raisin center" of the world?
   1. What natural conditions in the Fresno district favor the production of raisin grapes?
   2. What other factors have played a part in the development of the industry?
   3. Describe the seasonal work on a raisin ranch.

F. Compare the California date industry with that of the Old World.

G. How did California develop a fig industry? How does it compare with that of Asia Minor?

H. Summarize the reasons why California has become the leading commercial dried fruit producing region of the world.

V. Why is the dried fruit industry increasing in lands in the Southern Hemisphere?
   A. What districts in the southern hemisphere have climatic conditions similar to those of the Mediterranean lands of the Old World?
   B. Where in these lands is a dried fruit industry developing?
   C. Why is the industry developing more rapidly in Australia than in Central Chile and South Africa?
      1. Give reasons for its development in Mediterranean Australia.
      2. What fruits are produced? Why?
      3. What are the future prospects of the industry?

VI. How does world trade in dried fruits move?
   A. What are the chief exporting countries?
   B. What are the leading importing countries?
   C. To what countries does the dried prune pack of the United States move? Why?
   D. Trace the movement of dates, figs, currants, and raisins

from leading exporting countries and ports to chief importing countries.

VII. Summarize the conditions which cause the Mediterranean regions of the world to become the producers and exporters of dried fruits.

ACTIVITIES

1. Look up the most recent statistics on dried fruit production in the United States and other countries. You will probably have to locate these statistics under the various fruits. Make a table to show how the United States ranks in the production of the various dried fruits.

2. Investigate the following topics:

a. How the date palm was introduced into the United States.

b. Relation of insect life to fruit growing. Give specific illustrations.

c. The original home of the apricot.

d. The date industry in North Africa.

e. The raising of figs in the Near East.

f. The currant industry of Greece.

3. Vocabulary: Every industry has a vocabulary of its own. Do you know the meaning of the following words:

Air drainage	Humus
Alluvial fan	Nitrogen
Pollination	Evaporated fruits
Cover crop	Dehydrated fruits
Pruning	Fig Wasp

Ground Water

4. Map activities.

a. On a large outline world map, summarize the information you have gained from this unit concerning world production and trade in dried fruits.

b. On an outline map of California, indicate the leading producing areas of the following: dates, figs, raisins, prunes, pears, apricots. Locate Fresno, San Jose, San Francisco, and other important centers.

5. Collect and mount pictures which illustrate significant points in the dried fruit industry.

6. Collect labels and advertisements of dried fruits. Note the place and name of the marketing association. How does your collection check with the information given in this unit?

7. Additional problems for investigation:
   a. How have the California raisin and prune cooperative organizations been vital factors in the development of those two industries?
   b. How is California trying to solve her water supply problem? How is this problem related to the dried fruit industry?

References

Teachers' References

Baker, O. E., "Agricultural Regions of North America: Fruits and Nuts of California," *Economic Geography*, Vol. VI, (1930), pp. 278–286.

Bengston and Van Royen, *Fundamentals of Economic Geography*, pp. 485–489.

Blanchard, W. O., "The Grape Industry of Spain and Portugal." *Economic Geography*, Vol. V, pp. 183–193.

Bryan, *Man's Adaptation of Nature*, pp. 222–233.

California Fruit News, San Francisco, California.

California Dried Fruits Research Institute, "The Story of Dried Fruits."

Colby, C. C., "The California Raisin Industry, A Study in Geographic Interpretation," *Annals of the Association of American Geographers*, Vol. XIV, 1924, pp. 49–74.

Date Growing in Arizona, Bulletin No. 149, Agricultural Experiment Station, Tucson, Arizona, 1935.

Foster, Alice, "The Malaga Raisin Industry: A Study of Mediterranean Agriculture," *Journal of Geography*, Vol. 37, Jan. 1938, pp. 1–14.

Hodgson, R. N., "The California Fruit Industry," *Economic Geography*, Vol. IX, 1933, pp. 337–355.

Smith, J. Russell, *The World's Food Resources*, pp. 448–451, Grapes; and 417–421 and 425–427, Dried Fruits.

Torbert, E. N., "The Specialized Commercial Agriculture of the Northern Santa Clara Valley," *Geographical Review*, 26, 1936, pp. 247–263.

High School Texts and References

Aitchison and Uttley, *North America by Plane and Train*, pp. 200–207; 185–189.

Allen, N. B., *The United States*, revised edition, pp. 92–106.

Case and Bergsmark, *Modern World Geography*, pp. 158–170.

Crissey, F., *The Story of Foods*, pp. 341–357.

Jones, Clarence, *Economic Geography*, pp. 165–170.

Ridgley and Ekblau, *Influence of Geography on Our Economic Life*, pp. 362–369.

The Romance of the Date, Hills Bros. Co., 110 Washington Street, New York, a pamphlet.

The Story of Raisins, Raisin Growers Association, Fresno, California, a pamphlet.

Whitbeck, Durand, and Whitaker, *The Working World*, pp. 75–85; 436.

Keystone Units, Keystone Co., Meadville, Pa.

There is no end to truth, no boundary to its discovery and application.—Edward Everett Hale.

SOME WAR-TIME DEVELOPMENTS IN CHEMISTRY

B. S. Hopkins
University of Illinois, Urbana, Illinois

During the period of World War II chemical literature has been scant. This has been due in considerable measure to the fact that chemists have been engaged in defense work of various sorts or in investigations about which nothing could be said while hostilities continued. As the end of the struggle approached, some information was made public and now that the cease fire orders have been issued we are beginning to learn that much progress has been made while the curtain of military secrecy has been so tightly drawn. Many of the new developments will find a useful place in our peace-time living, although in some cases considerable time will be required to readjust and to adopt to the needs of a world at peace. But the abundance of new chemical materials, of which we are now beginning to learn, justifies the conclusion that chemists have been extremely busy during the war years and that chemistry will continue in the future to serve the varied interests of mankind even more effectively than it has been doing in the past. A brief review of some of the developments of the war years may be worth while.

I. The Metals

During the early months of the war there were few shortages which were more acute than those which we experienced in meeting the suddenly increased demand for the metals. Some of our essential metals, such as tin, manganese, chromium, nickel, and tungsten had been imported from other countries. The interruption of foreign trade interfered seriously with our domestic commerce. As the supply of one metal became limited, attention was directed toward some other metal which could be used as a substitute. So even those metals which we have in the past produced in abundance (iron, copper, zinc, lead, aluminum) became scarce as the demand rose beyond our ability to produce. As a consequence restrictions were placed upon the use of metals, substitutes were sought, increased production attempted, economy plans in the use of metals were devised and intensive salvage campaigns were instituted. Strangely enough gold and silver were about the only metals whose uses were not restricted by governmental action.

As a result of the combined efforts of prospectors, producers, refiners, fabricators, users and salvagers the metal shortage became less acute in 1944 and by the end of 1945 there was a sufficient supply of most metals for ordinary use. Tin continues to be scarce, a bare 12-month supply being available in July 1945. It is likely that the shortage of this metal will continue for a year or more. The supply is being conserved by the use of non-metallic food containers, of the low-tin solders and the continued employment of the economical electrolytic tin plate. Some other metals are still scarce but in some cases at least the shortage is due to lack of labor rather than to an inadequate mineral supply. Lead, antimony, bismuth, cadmium, platinum, and nickel are still short but as foreign commerce becomes more nearly normal, these shortages are not expected to be serious.

During the war period there were significant improvements made in the utilization of some of our mineral resources. Notable among these are the successful processes of obtaining manganese and chromium from our western ores which are low grade.[1] Improvement has been made in many metallurgical processes, and these are destined to contribute materially to the conservation of our natural resources for many years to come.

The light metals, aluminum and magnesium, and their alloys, have held a prominent position in the metal industries during the war period.[2] The sudden and enormous increase in the demand for these materials, especially in the aircraft industries created a frenzied effort to produce the metals in all possible ways. As a result enormous sums of money were spent in building plants which were to use new and untried raw materials with methods of extraction which obviously would not be able to meet peace-time competition. When the peak had been passed these "temporary" plants were closed down; some of the more promising were left in stand-by condition and others were declared surplus.

Aluminum was formerly extracted solely from bauxite ore, a hydrated aluminum oxide containing variable amounts of iron, silica and other mineral substances. The Bayer process for extracting alumina[3] requires material with a low percentage of silica. If the amount of silica exceeds 5–7%, increasing proportions of aluminum are lost in the form of insoluble silicates

[1] See the 1945 report of former Secretary of the Interior, Harold L. Ickes to President Truman.
[2] *Journal of Chemical Education*, Dec. 1942, p. 556.
[3] *Chemical and Metallurgical Engineering*, June 1942, p. 78.

which were formerly regarded as worthless. These tailings are colored red by the iron which they contain, and are familiarly known as "red mud." Since deposits of high grade bauxite are not very common, the first efforts to increase the production of aluminum were concerned with the use of ore containing a higher proportion of silica,[4] and from the "red mud" tailings from 30 years operation of the Bayer process. Since aluminum is found in many silicate rocks (feldspar, leucite, hornblende, garnet, zeolite, mica, koalin, clay), some aluminum was obtained from these sources.[5] But since these minerals usually contain a small percentage of aluminum and its recovery required a sintering and extraction, the process is expensive. Some success has been achieved with western clays.[6] and it is to be hoped that these methods will be able to compete with those depending entirely on bauxite.

New aluminum plants in the United States, built or contracted between January 1, 1940 and July 1943 cost about $600 million. Capacity at the peak reached 2.3 billion pounds per year. The capacity rate was attained for a few weeks near the end of 1943, but a decrease in production was soon made, because defense requirements had been over-estimated and lend-lease requests decreased sharply. The production of primary aluminum[7] in 1944 fell to 1.5 billion pounds and secondary aluminum added 600 million pounds. The wholesale price for primary metal of a minimum purity of 99% was 15 cents per pound.

It is interesting to note that the production of one pound of metal by electrolysis requires the consumption of 2 pounds of Al_2O_3, 0.8 pound of electrode, 0.1 pound of cryolite (or equivalent) and 12 KWH of electrical energy. It has been calculated that the aluminum industry consumed in one day more electricity than a city of 60,000 homes would use in one year. This high consumption of electric power is one of the main reasons for nation wide daylight saving during the war period.

Magnesium was formerly produced by the electrolysis of fused magnesium chloride from deep-well brines. This source of material quickly became inadequate when defense demands began to mount. Later magnesium chloride was obtained from

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[4] *Industrial and Engineering Chemistry*, Sept. 1945, p. 796.
[5] *Chemical and Engineering News*, Aug. 10, 1945, p. 1324.
[6] *Chemical and Metallurgical Engineering*, Dec. 1945, p. 108; *Chemical Industries 54*, 65 (Jan. 1944)
[7] *Industrial and Engineering Chemistry*, July 1945, p. 608.

ocean water[8,9] or prepared by the action[10] of hydrochloric acid on the oxide obtained from magnesite ($MgCO_3$), dolomite ($CaMg(CO_3)_2$) or brucite ($Mg(OH)_2$). One large plant at Lake Charles, La., used dolomite as raw material. It was roasted and the resulting mixture of CaO and MgO was fused with $CaCl_2$ in an atmosphere of CO_2. Magnesium chloride was then extracted leaving $CaCO_3$ as a residue. In another plant MgO was heated with coal in an atmosphere of chlorine and the resulting $MgCl_2$ extracted and electrolyzed. Several thermal reduction methods were attempted. In one, carbon was mixed with magnesium oxide and heated to a high temperature. The products were magnesium vapor and carbon monoxide. Two difficulties were encountered: (1) at intermediate temperatures a reverse reaction takes place; consequently the vapor of magnesium must be chilled quickly by flooding with very cold hydrogen or illuminating gas. (2) The air must be carefully excluded from the products of the process because of the danger of forming a highly explosive mixture. The most successful thermal reduction process uses ferrosilicon[11] as a reducing agent. The by-product is then a solid from which magnesium, of high purity, is separated by distillation. The Defense Plant Corporation constructed 13 plants for the production of magnesium metal. The total designed capacity was 586 million pounds per year. The production of primary metal[7] reached a peak of more than 41 million pounds in January 1944 and by the following month cutbacks were ordered. By December of the same year magnesium production had fallen to 17% of capacity. Three small government plants still continue to operate on account of important by-products and three privately owned plants, one operated by the Kaiser Corporation[12] and two by the Dow Company, at last reports were still operating. The production of one pound of magnesium by electrolysis requires 9.15 KWH of electric current. The lowest operating cost reported by an electrolyic plant was 11.51 cents per pound and by ferrosilicon reduction was 18.32 cents. The primary metal was sold under a government ceiling price of 20.5 cents. Secondary magnesium is easily recovered in pure form by distilling the magnesium away from other metals.[13] In 1944 nearly 30 million pounds of

[8] News Edition, *Industrial and Engineering Chem.*, Nov. 10, 1941, p. 1189.
[9] *Metals and Alloys*, Oct. 1944, p. 941.
[10] *Chemical and Metallurgical Engineering*, Oct. 1943, p. 98.
[11] *Chemical and Metallurgical Engineering*, April 1942, p. 87; Sept. 1942, p. 134.
[12] *Metals and Alloys*, July 1945, p. 102.
[13] *Light Metal Age*, *2*, 17 (July 1944).

secondary magnesium were obtained from scrap. It sold at 16 cents per pound. A cubic foot of magnesium costs less than the same quantity of any other metal except iron. During the war all the magnesium production was consumed by the armed forces. This has prevented the development of civilian uses. In the postwar period, peace-time uses for this relatively new metal must be found if the industry is to survive. Successful prewar uses, at least in the trial stage, include truck bodies, trailer busses, furniture, tools, cameras, typewriters and other equipment where lightness and strength are desirable. The manufacture of magnesium artificial limbs is attracting favorable attention since these are lighter, stronger and more economical even than those made from wood. A recent suggestion which may become very useful is to employ a bar of magnesium to protect underground iron pipes from the annoying and expensive effects of corrosion.[14] If a bar of magnesium is buried near the iron pipe and connected with it by a suitable copper wire, the magnesium, serving as the anode of the couple, is corroded while the iron pipe, the cathode, is not attacked by corrosive influences. It is estimated that such anodes will last about 10 years and certainly their replacement will be less troublesome and less expensive than the replacement of the iron pipe.

During the war period many of the less important commercial metals were studied intensively in order to increase their usefulness. Much progress has undoubtedly been made, but little has yet been published concerning these developments. As an example of the progress to be expected we may select lithium which has received unusual attention. In the pre-war period, the uses of lithium metal were restricted almost entirely to alloys, such as the German bahnmetal and scleron, which are said to contain less than one percent of lithium. The compounds of lithium have found some prewar uses in medicine, pyrotechnics, welding and soldering fluxes, air conditioning, enameling, and in increasing the life and capacity of alkaline storage batteries. The war caused a great increase in the demand[15] for lithium especially in connection with the air conditioning of ships and the storage batteries of submarines. As a result a process of lithium recovery was developed at Trona, California.[16]

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[14] *Chemical and Engineering News*, June 10, 1945, p. 984.
[15] *Chemical and Metallurgical Engineering*, Dec. 1944, p. 200.
[16] *Chemical and Metallurgical Engineering*, Jan. 1945, p. 156; Feb. 1945, p. 168.

By this process a difficultly soluble lithium sodium phosphate, Li_2NaPO_4, is recovered by flotation from the alkaline slimes of the soda recovery process from Searles Lake brines. The finished product contains up to 22% Li_2O so it is the richest natural source of lithium that we have. This new process has greatly increased our supply of lithium materials. The price of lithium metal in December 1945 was $12.50 per pound in large lots. The total domestic production of lithium minerals in 1944 was 14,600 tons, nearly 80% more than the production in 1943. A new use of lithium which seems to be of great interest to the engineering profession is the introduction of lithium metal in vapor form into the muffle furnace in which steel products are heat treated. The active lithium prevents carbon dioxide, oxygen and water vapor from forming surface scale or from removing carbon from the surface of the steel. The lithium compounds, formed by reactions with the gases in the muffle, are recovered and reconverted to the metal which is used again. The cost of this treatment is estimated at 75 cents per ton of steel. This is less than the cost of sanding, machining, finishing and polishing which are no longer needed when the lithium treatment is employed.

No discussion of war-time metals would be complete without at least a brief reference to iron and steel. For years these have been foremost essentials of nations both at war and at peace. Few factors were more influential in turning the tide of World War II in favor of the United Nations than two facts: (1) At the beginning of the struggle the allies controlled nearly three-quarters of the world supply of iron and steel; (2) the allies were able to expand their supplies of these materials to a much greater extent than were the axis powers. This expansion was particularly evident in the United States. The production of steel ingots and castings reached an all time high of nearly 90 million tons in 1944. This is 80% above the maximum annual output during World War I. This enormous total was reached in spite of the fact that a decline set in in March of that year. During the last half of the year the production was a half million tons below that of the same period of 1943. A significant development in this expansion is the fact that new producing areas are entering the scene. In 1945 production was reported from 27 states. Among the new producing states are Utah and Oregon, a significant development for the Pacific coast industries. If the American steel industry were operating at capacity

now it would be able to produce in 1.25 hours as much steel as the country made during the entire year of 1860.

We have justifiable pride in the splendid record which American industry has established in these years of serious conflict. There is, however, one sobering question which we must face in connection with the enormous strain upon our abundant mineral resources.[1] How long can our supply stand such a strain? In the case of iron and steel, good authority tells us that high grade iron ore, upon which our steel mills depend, will at the present rate of consumption be exhausted in 22 years or less. After that we must depend on low grade ores which will likely cut down on production and raise prices. The U. S. Department of the Interior estimates the time that our unmined supply of domestic minerals will last at the prewar rate of consumption (See Table 1). Of the 30 minerals listed only 8 have a life expectancy of more than a century while the supply of 12 others

TABLE 1

United States Department of Interior estimate of the unmined supply of domestic mineral resources based on the rate of consumption for 1935–39

	Years		*Years*
Magnesiums	Infinite	Gold	14
Salt	Infinite	Lead	12
Soft Coal and Lignite	4,300	Silver	11
Phosphate Rock	805	Bauxite	9
Molybdenum	422	Vanadium	7
Anthracite	195	Antimony	4
Potash	117	Tungsten	4
Iron Ore	111	Platinum	4
Sulfur	55	Mercury	3
Natural Gas	48	Asbestos	3
Fluorspar	40	Manganese	2
Copper	34	Chromite	1
Zinc	19	Nickel	−1
Petroleum	18	Tin	−1
Cadmium	16	Graphite	0

is estimated to be sufficient for less than 10 years. Even if these estimates are much short of the actual facts, they are sufficiently alarming to cause us serious concern. Nature has supplied us with an abundance of mineral resources, but they are far from being inexhaustible. It is clear that we must cease to be prodigal of our natural resources and begin at once a program which will include the preservation of our present supplies,

the conservation of our unmined minerals, the development of economical uses and the investigation of material which can serve as substitutes.

(*To be continued*)

SOUND MOTION PICTURE PROJECTORS FOR THE CLASSROOM

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Teachers who have seen the instructional sound films in science available today agree that they do a marvelous job of communicating understandings too difficult for the usual media of communication. However the task of bringing the audio-visual experience to the pupil involves more than getting the film you want when you need it. The problem of projection is perhaps the greatest barrier to a wider use of the sound film in classroom instruction. While science teachers are more accustomed to the use of all kinds of equipment than most others, even they find the weight and complication of present sound projectors obstacles to wider use.

Sound projectors can be much lighter and much simpler. Manufacturers tell us we have the present type machine because the customers have asked for a projector that would run forward and backward, show silent as well as sound films, show still as well as motion pictures, be usable as a public address system, have provisions for a phonograph attachment and serve in a large auditorium or in a small classroom. Hence we have the amalgamated assortment of plugs, switches and levers rivalling a Rube Goldberg creation. Extra weight goes with these extra gadgets and with the surplus sound output. If we as teachers can agree to limit our request to a classroom sound projector-period, we can have a twenty-five instead of a seventy-five pound projector and one with three instead of ten or more controls and have them at half the price.

It is my contention that the extra gadgets on present sound projectors are superfluous, especially for classroom use. I believe the silent instructional film is obsolete hence the extra speed control and troublesome two speed motor governor for silent films could be eliminated. The best instructional silent films today are the same sound films with the sound turned off

to allow for teacher or pupil explanation or for no verbal explanation at all. While the reverse feature on projectors is desirable the extra switches and troublesome spring belts involved plus the fact that reverse operation is often faulty, make this feature dispensable. The still-picture feature on a motion picture projector is another desirable feature that simply doesn't work, for either the film is burned or the picture is too dim. This job should be left to the more efficient filmstrip projector. There is no need for a public address system in the classroom, hence this feature is useless on a classroom projector. The phonograph attachment is rarely used. The ten or fifteen watt amplifier output needed for the auditorium is two or three times the amount needed for classroom use and it is here that most of the excess projector weight is found.

Simple, lightweight projectors of the kind described are a crying need in the field of visual instruction and we will have them only when teachers analyze their needs and insist upon the kind of equipment they need.

SPIDER PULLS HARD TO KEEP TRAP DOOR SHUT

A trap-door spider, that protects itself against intruders by securely holding down with its claws the door to its silk-lined underground dwelling, sometimes exerts a force 140 times its own weight to keep its door shut, states Dr. Walker Van Riper, curator of spiders of the Colorado Museum of Natural History. For a 150-pound man the equivalent would be 21,000 pounds, or over 10 tons, Dr. Van Riper estimates.

Usually the only sign of a trap-door spider's nest is a hair-thin crescent line half an inch or so long. To keep its door closed, the spider grasps the door around its circumference with its tarsal claws, wedging her body across the passageway for anchorage.

To measure the strength of a particularly vigorous "holder-downer," Dr. Van Riper drilled two holes in her door and threaded through them a loop of string. When the spider raised the door about an eighth of an inch, he measured the spider's pulling power by lifting with spring scales the string attached to the door. A force of 14 ounces was indicated, he reported in *Natural History Magazine*.

AIRPORT USES NEW LIGHTING AID

Vertical walls of light shooting skyward from the lateral edges of the runway will provide an "aisle of light" for pilots landing aircraft during soupy weather in Idlewild Municipal airport, New York City's gigantic new landing field. This innovation, developed by engineers of the Westinghouse Electric Corporation, will be one of several landing aids designed to make Idlewild an all-weather haven for pilots of transcontinental and world-flying aircraft.

ARITHMETIC TAUGHT AS A BASIS FOR LATER MATHEMATICS

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Many years ago Felix Klein suggested that a real understanding of the basic concepts of arithmetic anticipates and paves the way for the subsequent learning of secondary and higher mathematics. It is the purpose of this paper to discuss briefly some specific arithmetical concepts which can contribute to later insight and appreciation of fundamental mathematical ideas.

The arithmetic of the elementary school should deliberately be taught as a system of connected ideas rather than as a collection of skills. The meaning of number, the fundamental operations, arithmetical notation, the basic processes should all be developed and explained in *general* terms. They should be rationalized in a manner consistent with the maturity of the learner. As Morton and others have pointed out repeatedly, the aim should be to encourage growth in thinking at steadily rising levels of abstractness. "Only when arithmetic is taught in terms of the meaning theory can there be reasonable assurance that the pupils' experiences will be rich in opportunities for the cultivation of the higher mental processes."[1] Such an approach will go far toward assuring a somewhat more rigorous treatment of elementary mathematics beyond school arithmetic. The principal areas in which more generalized treatment might be undertaken would include: (1) the nature of number; (2) the nature of operations; (3) the nature of relationships; and (4) the language used in mathematics.

The Nature of Number. Underlying the very notion of number, of course, is the idea of counting, in which children engage even before entering school. Here the learner takes one of his first steps in abstraction. He makes use of the quality of mind which enables him to relate things to things, for the essence of counting is establishing a one-to-one reciprocal relationship between the items in two groups of objects, or the elements of two sets. Children find this matching process simple and natural. Thus to count an aggregate, or a set, means to identify the set as to its numerical quality with another set, usually one that is more familiar. It is not essential to have number words or number symbols to determine the equivalence of two sets.

[1] Morton, R. L. *Teaching Arithmetic in the Elementary School*, vol. III, p. 16.

Whenever the elements of two sets are matched in this way, i.e., when each element of one set is paired with just one element of the other, and each element of the second set is paired with just one element of the first set, the two sets are said to have the same cardinal number, or the same potency; they are cardinally similar, or equivalent sets. Thus the recognition of cardinal equivalence involves stripping the objects or elements of all properties save their *distinctiveness* or *uniqueness*. The crucial idea is that the cardinal number of a set is what is common to that set and to every other equivalent set. This idea is of a high order of abstractness, and, as history amply testifies, one that was embraced by mankind only with considerable effort.

Closely related to the idea of cardinal number, yet distinctly different from it, is the notion of ordinal number. In essence, it has to do with particular sequence, or order. The idea of cardinal is an *outcome* of the act of *ordering*, or counting off; the last ordinal used, which tells the result of the count, is then given a cardinal meaning. When we have labelled, in order, the *eighth*, we have labelled *eight*. Every element in a set has ordinal significance; no element has any cardinal significance. The cardinal number of any set of finite things is the same in whatever order we count them. The appreciation of such ideas is essential to further insight into the nature not only of number and of numeration in arithmetic, but also of more advanced concepts in mathematics, such as the number concept in algebra, the notion of the continuum, the concept of the infinite, the theory of assemblages, and so on.

In connection with an understanding of the meaning of number, there are several other related ideas, the appreciation of which would serve to point toward better mathematical understanding later; namely, the rôle played by symbols, the meaning of zero, and the significance of positional value.

Consider the rôle played by symbols—both number-words and numerals. As symbols, they again represent a highly abstract concept. The realization that *3*, / / /, *three*, *trio*, and *trey* are synonymous symbols and represent the three-ness of many groups, irrespective of any characteristics of the elements save distinctness, or of the order in which they are considered, is by no means an easy concept to grasp. It is, however, analogous to the use of literal symbols in formulas such as $T = D/R$, $A = lw$, or $I = Prt$. The thinking required in both cases involves, among other things, an appreciation of the *generality of symbols*. In the

one case the symbol, a constant, represents *all* the sets that are cardinally equivalent; in the other case the symbol, a variable represents *all* the possible values in the variable's range. Mathematical symbols are characterized by their abstractness, their generality, and the further fact that they may be chosen arbitrarily.

A word about the symbol zero. By the time a pupil has achieved reasonable mastery of arithmetic, he should appreciate that this particular symbol has several meanings. It may play the rôle of a place-holder (positional value), as when used to distinguish 23 from 203, etc.; or it may be used as a cardinal number to designate an empty set, or null-class, as when the answer to the question "how many?" is "none"; or it may play the rôle of an ordinal number, the origin or reference point, as when used on the scale of real numbers, $\cdots -3, -2, -1, 0, +1, +2, +3, \cdots$. Clear understanding of such meanings not only makes for more effective use at the time they are being learned in arithmetic, but also lays a good foundation for future mathematical learning.

Finally, we come to the concept of positional-value, that simple yet powerful device, greatly augmented by the Hindus when they invented the zero. The positional notation, despite its simplicity, is a highly refined artifice for representing every number as a sum, expressed by a row of digits. Each digit represents a product of two factors: one factor, the intrinsic factor, indicated by the *digit itself*, and the other, the local factor, indicated by the *position of the digit* in the row. The local factor is, of course, a power of the base: for units' place, b^0; for tens' place, b^1; for the first place to the right of the zero or unital point, b^{-1}; etc. Thus in the arithmetic of the grades it is sufficient that the pupil *understand* that 354 means "3 hundreds+5 tens+4 ones," or $3(100)+5(10)+4(1)$; later he should come to appreciate that 354.68 means $3(10)^2+5(10)^1+4(10)^0+6(10)^{-1}+8(10)^{-2}$.

The Nature of Operations. Insight into the nature of operations presumes an appreciation of the meaning of the symbol for equality ($=$). The notion of equality arises early in the experiences of childhood, and before long the pupil senses that the essence of equality is that it permits of *substitution*. Sooner or later, as he advances in mathematical maturity, he ought to realize that the relationship of equality is not only *reflexive* (xRx), but also *symmetric* (xRy implies yRx) and *transitive* (xRy and yRz implies xRz). Of course, he will not, as an arithmetic

pupil, learn these technical terms. He should, however, appreciate that a quantity is equal to itself ($a \equiv a$); that if $a = b$, then $b = a$; and that if $a = b$ and $b = c$, then $a = c$. Eventually he will come to understand that not all relationships possess all three of these qualities.

In this connection, it is to be hoped that arithmetic teachers will in the future put more emphasis on the notion of a "mathematical expression." An expression results from the representation of one numer by others with symbols of combination and operation. If we enclose an expression in a parenthesis, no matter how complex the expression, it may be operated upon as if it were a single symbol representing that number. It is always permissible to substitute a single symbol for any expression whatever.

Perhaps the first operation learned by children is that of addition. The abstraction involved here is that number exists as an independent concept; that although, strictly speaking, there is only one number 4, we may properly think of combining 4 with 4 so as to retain the units unaffected, even though the "fourness" disappears in the sum 8. In short, units of numeration are completely interchangeable. The essence of addition is finding one number which contains in itself as many units as the given numbers taken together.

Because of the invariance of the cardinal number of a set, that is, because the cardinal is unaffected by the order of counting, it follows that a sum is independent of the order in which the several sets are added. Here again, the learner's conceptual understanding grows with experience, and as he proceeds, he should appreciate, without the use of technical terms, of course, that the operation of addition is *commutative*, or $a+b=b+a$; also, that it is *associative*, or $(a+b)+c=a+(b+c)$. He learns this first by thinking about "groups"; but, with an eye toward later mathematics, he should soon become familiar with generalized numbers (i.e., literal symbols) and parentheses, chiefly through the use of simple formulas.

The operation of multiplication becomes possible by virtue of the fact that an indicated sum in which all the parts are equal is determined once we know one of the equal parts and the number of parts. The pupil soon learns that the part repeated is the multiplicand, and the number which indicates how often it occurs is the multiplier. He should understand that multiplication is *commutative*, or $ab=ba$; also, that it is *associative*, or

$(a)(bc)=(ab)(c)$. But the arithmetic teacher of the later grades might very well stress the idea that multiplication and addition may be combined according to the *distributive* law, i.e., $a(b+c)=ab+ac$. This is not nearly as formidable as it seems; formulas such as $P=2(l+w)$ and $A=h/2(b+B)$ may well lead to such understanding. Pupils should be able to think of 3×24 as equal $3(20)+3(4)$, or $3(20+4)$.

As the pupil matures, he should be able to think of a product as the number related to the multiplicand as the multiplier is related to the unit, or, $ab/b=a/1$. Eventually the pupil should come to regard multiplication as a unique operation (connected with addition only by the distributive law); namely, as the process of finding from two elements, p, q, an element unequivocally determined, pq, called "the product of p and q," which by commutativity equals qp. The two elements are called the *factors*, and the distinction between multiplier and multiplicand, for many purposes, is entirely irrelevant, as for example, in *volts*$\times$*amperes*, or $PV=kT$.

The two operations of addition and multiplication, in their simplest application, are tantamount to the problem: from two given numbers to make a third number. The inverse of such a problem is, of course, given the result of a direct operation and one of the components, to find the other component. Subtraction and division are inverse operations, respectively, of addition and multiplication. Thus the inverse of addition is to solve the conditional equation $a+x=b$, where a is given as one of the components, and b is the given sum. The component to be found, $b-a$, is called the difference between b and a. By assuming the "law of signs," we have: $(b-a)+a=b-a+a=b$, and $a+(b-a)=a+b-a=b$. Thus subtraction is commutative and associative.

Mathematically, division has two meanings: (1) the inverse of multiplication, and (2) remainder division. In accordance with (1), division simply means *finding one of two factors when their product and the other factor are given*. The factor sought is the quotient, q; if the given product is a and the given factor is b, then $a/b=q$. If $a=0$, then $q=0$. If $b=0$ and $a\neq0$, then the operation of division is meaningless and impossible; if $b=0$ and $a=0$, then q is *indeterminate*, i.e., any value for q would satisfy the equality $a=bq$. It is also clear that $a/1=a$.

The second meaning of division involves the notion of *regrouping a given set*. In other words, given the dividend a and the divisor b, where $a>b$; the purpose of remainder division is

to convert a to the form $bq+r$, where $r<b$, $b\neq 0$. The quotient q and the remainder r are both integral; r may or may not be equal to zero. Remainder division $(a \div b)$ answers two questions: first, what multiple of b when subtracted from a gives a difference less than b? and secondly, what is this difference or remainder?

Suppose I have 17 oranges, and consider, in turn, these two problems:

(A) If I put 3 oranges in each basket, how many baskets will I need?

(B) If I distribute them equally in 3 baskets, how many oranges will there be in each basket?

These two problems represent the two ways of regrouping a given set which constitute remainder division. In Problem (A), we regroup the given set $(=17)$ into smaller sets each with the same cardinal as a given set (the divisor $=3$), and a remaining set $(=2)$ whose cardinal is less than that of the divisor. Here the quotient is the number of equivalent subsets. Since there is no implication that the original units (elements) are equal in size, this process is not the same as measuring. It should be pointed out, however, that division, when used in this type of problem, is often referred to as "the measurement idea" of division. In Problem (B), the given set $(=17)$ is regrouped into a given number of equivalent subsets (the divisor $=3$), with a number of units (elements) remaining; the cardinal of the remaining group is less than that of the divisor. In this type of problem, the use of division is frequently called "the partition idea" of division, although from a mathematical standpoint, it is the same operation as before. In short, both problems illustrate applications of the same mathematical concept, namely, remainder division, or regrouping a set into subsets.

When teaching remainder division in elementary arithmetic, these two types of problems should be stressed, as well as these two aspects of the division operation. That none of the technical terms should be introduced goes without saying. However, it would seem desirable that eventually the learner should begin to appreciate the distinction between division as the inverse of multiplication and division as the regrouping of a given set.

The Nature of Relationships. As Keyser has somewhere pointed out, "to be, is to be related." Quantities, as such, have meaning only when seen in their relation to other quantities. Several ideas encountered in elementary arithmetic contribute

potentially to more subtle concepts of later mathematics. Among these are the notions of (1) comparing quantities by subtracting; (2) comparing quantities by dividing; (3) a rate; (4) dependent quantities; and (5) an average.

One of the first methods by which a child learns to compare two quantities is by difference; that is, by finding how much more (or less). To do this, he subtracts. This, of course, is commonplace. But the concept is used over and over again in later mathematics: for example, when interpolating; when finding deviation measures; in the familiar $x+\Delta x$ and $y+\Delta y$ of beginning calculus; in the theory of finite differences; etc. Later he learns to compare two quantities by dividing, which leads to the concept of ratio. For many pupils this is a difficult idea to grasp. A likely reason for the difficulty of the ratio concept is the fact that the rationalization may not be adequate. In any event, a clear understanding of the meaning of ratio is indispensable in later mathematics, as for example, when studying the metric properties of geometric figures, inverse variation, trigonometric functions, the derivative as the limit of $\Delta y/\Delta x$, limits in general, infinite series, etc.

The idea of a rate, such as miles per hour or grams per cubic centimeter, will of course be encountered frequently in later work. It involves other related concepts of basic significance, such as unit, dimension, measurement, precision, error, and so on. The treatment of the topic of measurement might well be improved in secondary mathematics; the groundwork for it should be carefully developed in the earlier arithmetic.

Perhaps one of the most fruitful concepts which school arithmetic can begin to build early is that of dependent quantities, or related variables. Many illustrations may be used: the total cost to the unit cost and the number of articles; the distance to the time and the rate; the percentage to the base and the per cent (or rate); the interest to the principal, the time, and the rate of interest. Many of the simple mensuration problems involve such functional relationships: $P=4s$; $A=s^2$; $A=lw$; $P=2(l+w)$; $C=2\pi R$; $A=\frac{1}{4}\pi D^2$; $A=\frac{1}{2}Bh$; $V=lwh$; $V=e^3$; $V=\pi R^2h$; etc. Relationships between variables should be taught by means of tables, graphs, and formulas. The *generality* of the symbols in formulas should be stressed. The generality of the relationships should also be emphasized. The thinking should be general: thus, if the side of a square is doubled, what does the perimeter become? the area? If the length of a rectangle is

doubled, while the width remains constant, what does the area become? Many other such practice exercises should be included in the instruction. Finally, in developing the ability to solve verbal problems, the recognition of relationships between quantities should be emphasized as the basis of success.

The Language Used in Mathematics. From the very beginning, pupils should gradually come to appreciate the power and conciseness with which certain terms express important and useful mathematical ideas. Clearly such terms should not be introduced until the learner has had some experience with the idea or concept in question. But when the need for the term arises, when it becomes apparent that both economy and sharpness of thinking will be served thereby, the specific term should be presented, and from that time on, the term should be employed by both teacher and pupil consistently whenever its use is warranted.

Such terms may be thought of for convenience as falling into several categories. For example, certain words contribute significantly to rigorous thinking, whether mathematical or not, especially when the thinking is in general terms. Such words might include, among others, the following:

arbitrary	*hypothesis*	*random*
assumption	*induction*	*reciprocal*
axiom	*inference*	*rigorous*
consistent	*initial*	*sequence*
converse	*inverse*	*simultaneous*
deduce	*notation*	*successive*
eliminate	*origin*	*terminal*
empirical	*proposition*	*unique*

It should be noted that virtually every one of these terms is commonly used in ordinary discourse, quite apart from its specific mathematical connotation, to which, however, the nontechnical meaning is more or less closely related. The arithmetic teacher, or the ninth-grade mathematics teacher, may well ask, how many of these twenty-four terms can be reasonably introduced in connection with the arithmetic of the upper grades, or the general mathematics of grades 7, 8 and 9? It would seem that many of them could be brought in without stretching a point. For example, the meaning of *converse* could be illustrated by the fact that if a right angle is an angle of 90°; then any 90°-angle is a right angle; or, if two lines that are parallel never intersect, then two lines that never intersect are parallel; etc. In short, any proper definition is in effect a proposition whose con-

verse is true. Again, when explaining the meaning of an angle in terms of rotation, the positions of the sides may be characterized as *initial* and *terminal* positions. When explaining the meaning of *pi* (π), the expression *unique* value may easily be defended. Or when teaching the properties of a circle, the center is a point *unique* among all the points within the circle. When discussing formulas, the terms *notation* and *successive* values might be employed. In connection with graphs, one might well refer to the *origin* of the number scale, to pairs of *simultaneous* values, etc.

A second category contains useful basic terms which refer to metric and quantitative relationships:

accurate	*excess*	*rate*
approximate	*graduated*	*ratio*
deviation	*increment*	*residue*
discrepancy	*interval*	*scale*
equivalent	*period*	*standard*
error	*precision*	*tolerance*
estimate	*proportional*	*unit*

Closely related to these terms are others which occur in connection with geometric concepts and spatial relationships:

acute	*inclination*	*similar*
arc	*oblique*	*slope*
coincide	*obtuse*	*symmetric*
congruent	*parallel*	*translation*
converge	*perpendicular*	*transpose*
direction	*revolution*	*transverse*
diverge	*rotation*	*vertical*
horizontal	*segment*	*gradient*

Still another group of terms, nearly all of which are essential to meaningful learning of elementary mathematics, would include:

average	*exponent*	*limit*
coefficient	*expression*	*maximum*
constant	*extrapolate*	*minimum*
continuous	*factor*	*multiple*
coordinate	*function*	*power*
dependent	*identical*	*relation*
determine	*indeterminate*	*terms*
discrete	*interpolate*	*variable*

A final category would be comprised of basic technical and scientific terms, such as the following:

acceleration	*expansion*	*rate of change*
axis	*force*	*resistance*
balance	*instantaneous*	*speed*
component	*mass*	*tabulate*

concave	*periodic*	*uniform*
contraction	*pressure*	*vector*
convex	*projection*	*velocity*
density	*reaction*	*volume*

With this array of terms before her, the typical arithmetic teacher will doubtless arch her eyebrows or shrug her shoulders, if she does not, indeed, throw up her hands in horror. We have said and done so much in recent years to lighten the vocabulary load in arithmetic; now it is implied that a hundred or more rather formidable terms be reintroduced! To begin with, it may be pointed out that in easing the vocabulary load, it was not these words, in the main, that were discarded, but other far less useful expressions, such as denominate numbers, least common multiple, highest common factor, aliquot parts, involution, evolution, and others. In the second place, what is suggested here is not that all these terms be taught for mastery, but that some of them should find their way, gradually, into the pupil's working and reading vocabulary, so that when he encounters them (or the ideas which they connote) his understanding of both the idea and the term will mutually clarify and deepen his mathematical insight and appreciation.

In the third place, it is contended that an understanding of these terms will unquestionably enhance his capacity to think quantitatively and critically about matters that are not primarily mathematical. Such understanding is thus of potential value to all pupils, whether they continue their study of mathematics or not. What would seem to be eminently desirable as well as reasonably attainable is some anticipation of more mature and effective thinking, keeping pace, to be sure with the pupil's growth. For those who will continue their mathematics, this is not asking too much.

Finally, it is urged that success depends to a very considerable extent upon the manner in which these terms are introduced and incorporated into the learner's equipment. It is not suggested that these terms be made a prominent feature of the instruction, or that they be treated in a formal way. Their use should be functional. The terms should be introduced in a natural way, and deftly woven into the pupil's thinking. Upon initial introduction, they should be deliberately related to other familiar words, ideas and experiences. For example: *acute* means sharp—as, an acute pain; *obtuse* means dull—as, an obtuse person; *slope* suggests hillside; *arc* suggests arches and archery; *axis* suggests

axle; *horizontal* suggests the horizon; *notation* suggests notes, notebook; *initial* suggests the first letters of one's name; *terminal* suggests a bus terminal; *uniform* suggests the apparel worn by policemen, soldiers and sailors; *mass* suggests massive. Pupils doubtless will be able to associate *identity* with identifying oneself; *assumption* with assuming; *tolerance* with tolerating or accepting; *proposition* with proposing; *transverse* with transcontinental or trans-Atlantic; and so on.

Furthermore, it is often possible to use, temporarily, some familiar, homely, informal word in lieu of the ultimate precise term. Although it conveys the idea but crudely and approximately, it nevertheless insures the desired transition in thought. For example: we may say "helter-skelter," "here and there and everywhere" before using *random;* we may say "beginning point" or "starting place" in lieu of *origin;* "level" instead of *horizontal;* "even" or "regular" instead of *uniform;* "ends" or "winds up" for *terminates;* "divided" for *graduated;* "exactly alike" for *congruent;* "suppose" or "make believe" for *hypothesis;* "roughly" or "about" for *approximate;* "guess" or "imagine" for *estimate;* etc. Such facility with words challenges the teacher and stimulates the pupil. It is well-worth the effort required. The thoughtful teacher will recall Kasner's prize definition of mathematics as *the science which uses easy words for hard ideas.*

AN APPARATUS FOR DEMONSTRATING ADSORPTION BY CHARCOAL

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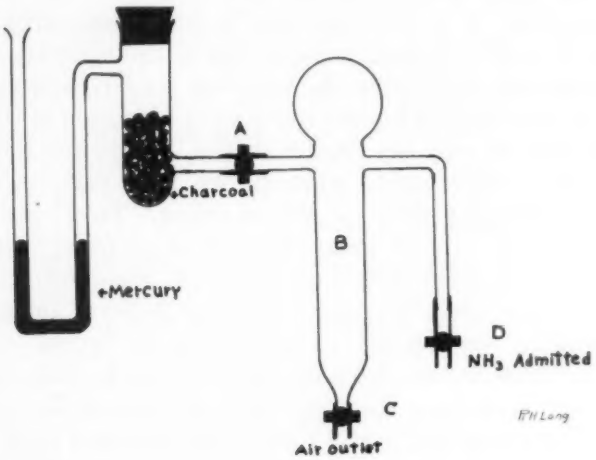
Because of the difficulty in demonstrating the property that charcoal has of adsorbing gases to an entire class of chemistry students by the conventional methods, the apparatus as shown in the diagram was devised and found to be very useful, because the change in the mercury column could be readily seen.

The size of the apparatus will depend on the number in the class. It can be made from tubing of the sizes ordinarily used in the laboratory, while the work can be done with a common glass-working burner. The apparatus can, if necessary, be made from sections of glass tubing and rubber stoppers. Pinchcocks on sections of rubber tubing are used in place of stopcocks.

After the charcoal has been placed in the proper tube and the mercury added, pinchcock *A* is closed and those at *C* and *D* are opened. Tube *B* can be filled with ammonia by heating a test tube containing concentrated ammonia water and connected to the apparatus at *D*. A piece of moist red litmus paper held at *C* will indicate when all of the air in the tube has been replaced by the ammonia. Pinchcocks *C* and *D* are then closed



and *A* is opened. In a short time the mercury will rise in the inside tube as adsorption takes place. Before starting the demonstration the apparatus can be clamped to a stand in full view of the class. Note: If the tubing used for the mercury is of small diameter, a piece of wire inserted into the side of the tube where the mercury is poured will prevent the formation of small air pockets.

MORE BEES NEEDED

Wanted—more bees! The number of bee colonies should be increased as much as 8% this year to help provide more adequate pollination of legume forage crops, fruits and vegetables, the U. S. Department of Agriculture states.

A greater pollinating burden is placed on honeybees today than every before as bumblebees and other former pollinating insects have been decreasingly rapidly.

At least 50 crops depend upon bees for pollination or yield more abundantly when bees are working.

The activity of bees as polinating agents is conservatively estimated to be 10 to 20 times more valuable than the honey and beeswax they produce.

WHAT INDUSTRY EXPECTS A STUDENT TO GET FROM A HIGH SCHOOL PHYSICS COURSE*

T. LEE, JR.

Assistant to Vice-President, Personnel, Training, United Air Lines

One of the most enjoyable ten minutes of my life was spent on a ferry boat between Oakland and San Francisco. This happened before the Oakland-San Francisco Bay bridge was built. I was at that time making a study of questions made by children.

On that trip I met little Johnnie, who was seven years of age. I happened to overhear him ask his mother, "Mom, why do the big open pipes (ventilation pipes) at that end face the way we are going and those at this end face in the opposite direction?" His mother answered, "Johnnie, don't ask so many questions."

As we walked from the back end of the ferry to its front end, Johnnie asked six intelligent questions, to which he received the following information, "Johnnie, don't ask so many questions." Johnnie was curious, was interested in the things he saw. Interested mainly in functions, the why and the how of things. Johnnie was also unperturbed by his mother's one answer, "Johnnie don't ask so many questions." Johnnie's eye saw a seagull overhead, keeping up with the ferry without flapping its wings; soaring in the upward pressure of the air coming over the bow of the ferry. "Mom, why doesn't this seagull flap its wings, like that sea-gull over the water does?"

I could not stand the unanswered intelligent queries of a growing boy. As a stranger, I offered an answer. During the next ten minutes, I tried to answer a score of questions that came in machine gun fashion. Someone in society had responded to Johnnie's queries, and he was going to make the most of the opportunity. The inhibitions, which his mother unconsciously was building up, did not affect Johnnie.

As human beings, there is only one thing of which we are absolutely certain, i.e. "change." Each one of us will change. The persons with whom we associate will change. Social groups will change. Governments will change.

It is said that truth and fundamental laws do not change.

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* Read at the Physics Section Meeting of the Central Association of Science and Mathematics Teachers at the United Air Lines Office, November 23, 1946.

Most of us will agree with that statement. But all of us will agree that our interpretations of truth and fundamental laws have changed, and will change.

We learned in our youth that there were 92 chemical elements. But all of them had not been isolated or produced at that time. Several centuries ago there were some progressive thinkers who stated that it might be possible to change base metals into precious metals. We refer to them as alchemists but society of their day called them crazy. In some cases society treated them rather roughly.

Today neuclear physics has certainly changed our interpretation and attitude towards the alchemist. For the alchemists dreams came true. Base metals have been changed into precious metals by our physicists. We no longer have one hydrogen or one uranium. The physicists have given us a new word "isotopes" to interpret a fine shade of difference. For example we have uranium with several atomic weights, or isotopes. If Johnnie were old enough, he would ask, "How many of the basic elements have isotopes, and how many classifications of isotopes for each element?"

When Johnnie has asked, probably a million questions, and has mastered the reasons for those physical things which he can actually see with his eyes, it is hoped, that his imagination will begin to see isotopes and the like. It is also hoped that he will get from his physics teacher an answer, not an inhibitory evasion, as Johnnie's mother answered. When Johnnie has begun to see isotopes and the like, he will be ready to begin to go places mentally.

What is education? Is education the teaching and the checking of the knowledge level of a list of facts? Or, is education the development of a wholesome mental attitude towards the phenomena of our earth, towards life, as lived with human beings, individually and in social groups?

May I divide education into three phases.

First, a wholesome mental attitude. Johnnie needs a wholesome mental attitude, towards a single individual as a human being, and towards society. Society has many sub-divisions. Some are motivated by loving cooperation, but some are often motivated by organized selfishness, hate, malice, which lead to individual conflict or group wars. He must learn to interpret the phenomena of this earth, so that he will not be afraid of natural events.

The second phase of education; the selection of a limited field. Johnnie's life is too short to master all the details of the whole of life, even if he had the desire or aptitude to accomplish it. The various social groups of society must help Johnnie to find some particular field or segment of life in which he is particularly interested and for which he possesses an aptitude.

Fortunate is that person, who upon graduating from high school, finds that his interests and aptitudes have selected the same field.

Unfortunate is that person whose education did not create within him a mental attitude concerning or interest in some field of life for which he possessed some aptitude.

The third phase of education comes after a wholesome mental attitude towards life has been created, and, after an aptitude has been demonstrated for a particular endeavor or field of life. The third phase of education is the transfer of knowledge and experience in a particular endeavor. This is of vital importance, but should not be more vital, than the first two phases.

It is not my desire to create an attitude in you that Industry should pass judgement on how well these three phases of education are being accomplished. But I would like to state, that Industry does have many young people who are seeking employment who have graduated from High School, without a wholesome mental attitude, without any particular interest in or aptitude for a particular field of endeavor. They are just seeking a job, any job.

If no interest exists, or, if no interest can be created in a particular job in industry, the odds are that the employee will not be efficiently productive, or happy in his work.

Pride in oneself, pride in one's work, pride in one's industry cannot be created overnight. They are the results of a wholesome mental attitude created by society, i.e., the home, the school, the church, the gang or social groups.

Some industries in the past gave very little thought to whether the prospective employee had a wholesome mental attitude and aptitude. It was the era of "brawn or beef," not "brains." In our industry, Air Line Transportation, we need very little "brawn or beef" but we do need brains. For the reason that so many of the classifications of work are very technical in nature.

In United Air Lines there are over 500 classifications of work, most of them very technical, but all having one aim; to give safe, speedy, and pleasant transportation to passengers, and

fast service for cargo. Since our employees are distributed across the United States, and are constantly representing our company in the community in addition to performing their technical duties, our company gives much thought to the three phases of education: A wholesome mental attitude, the evidencing of interest in and aptitude for a chosen field of work, and a general knowledge and experience in the chosen field.

Ten years ago my friend Johnnie was seven years of age; when I met him on the ferry. To our joy, we found that we lived only a few blocks from each other. In a few days Johnnie came to visit me. Johnnie always asked questions. He wanted to know "why" and "How" things function. Johnnie never used the word function. It was "why" or "how." Johnnie was a normal boy. He built things, first, small models of gliders, then, larger models. He got interested in radio, so I gave him a code set. Before long he could send and receive coded messages. One Christmas he got a model airplane gas engine. Before long he was flying his engine powered model airplane. What satisfaction and happiness Johnnie received in succeeding. The more he succeeded the harder he worked. The harder he worked in his play, the more he had to learn, to succeed, in the next venture. Johnnie didn't know it but he was learning industry. He was adding to his wealth. By wealth, I do not mean dollars or capital. The definition I like is: Wealth is those energies, facilities, habits contributing to make people industrially efficient.

As soon as the Germans began to use V-1 bombs driven by a jet motor, Johnnie asked himself the question, "Can I drive my powered model airplane with a jet motor?" Also, he thought, "Who can help me?" He found the information he needed. He built his jet motor and flew his model. He was the first boy in his neighborhood to fly a jet model airplane. Maybe I should have stopped, when I said, that he was the first boy in his neighborhood period.

Who helped Johnnie? His father died when he was six. His mother helped in many ways, but gave little help in answering his questions. The neighbors helped, the Boy Scouts, the Church, and last, but not least, the school teachers.

Johnnie has a wholesome mental attitude towards human relations. He practices democracy. He is becoming cultured. He has a mental balance between health, work and play. He knows the value of a dollar as well as the qualities that make a man wealthy without having a million dollars. Johnnie graduates

from high school this year. He is a normal healthy American boy.

Mr. Tapley has asked me to discuss, what Industry expects a student to get from a physics course.

Industry expects a high school graduate to have a wholesome mental attitude towards life with its many ramifications. The physics teacher has a definite share in helping to create this wholesome mental attitude.

The physics teacher has also a share in helping in the evaluation of the student to see, if there exists an interest and aptitude for some field of endeavor, which may require a fuller knowledge of basic physics.

The physics teacher shared with the other teachers the responsibility for creating a wholesome mental attitude and for the selection of some field of endeavor. But the whole responsibility rests with the physics teacher for the third phase of education, i.e., imparting a fuller knowledge and experiences of basic physics.

I do not intend to imply that every high school student should be given a full knowledge and experience of basic physics. Somewhere, during high school evaluation should separate those without an interest and without an aptitude for physics from those with an interest and aptitude for physics.

I believe that all physics teachers would be very happy to have this evaluation made prior to the advance basic high school physics.

A moment ago, I spoke of the knowledge and experiences of basic physics. I use the words knowledge and experiences synonymously with "why" and "how." The transfer of work knowledge, which can be memorized and written in answer to questions is not necessarily desired by Industry. Industry is not interested in semantics, as such, but is interested in the functions of the fundamental laws of physics, as they can be applied in everyday work life.

Mr. Tapley stated to me that there are twenty different ways of writing or expressing division. The airline cargo-handler, while loading cargo in an airplane, is interested in dividing the load, some to go into the nose and some to go into the tail of the airplane. The load must be divided in accordance with a fundamental law of balance. An Air Line in this case is interested in word "divide" only insofar as the division of the cargo keeps the airplane in the proper aerodynamic balance. Yes, the em-

ployees responsible for loading cargo should have had a physics course.

May I enumerate a few of the 500 classifications of work on an air line, stating some of the fundamental laws of physics which are required? The pilot needs to know fulcrums, levers, transfer of energy, thrust, drag, vibration, electricity, radio, instruments, navigation, celestial navigation, meteorology and etc.

May I give to you the experience of a friend of mine. He was in the Air Force. He received orders to pilot a plane from Washington, D. C. to Florida, then across the south Atlantic. He received his orders 5 p.m. Monday. He left 9 a.m. Tuesday, flew to Florida picked up his cargo and a new crew. The next night they were over the Atlantic. The navigator had just graduated from an Army Navigation school. The Pilot and Navigator were strangers to each other, and strangers, also, to the celestial bodies in the southern Hemisphere. They had their maps and navigation equipment, their knowledge and experiences, and a strange set of stars. They plotted their course, made constant checks and successfully flew across the Atlantic. They possessed knowledge and experience. They had learned to apply knowledge in a practical manner. They had succeeded before. They succeeded this time because of functional application.

The Maintenance Mechanic of an airline services and checks the equipment on an airplane. Before the plane flies away the mechanic has signed the maintenance form, stating that to his knowledge the plane is airworthy. The lives of passengers depend upon his judgment. The air line must know that the mechanic not only has the knowledge but can apply that knowledge. The company knows also that the judgment of the mechanic can be relied upon for it has been evidenced time and again, before the mechanic was assigned to the responsibility of signing the form of airworthiness.

The Line service Mechanic needs to know materials, hydraulics, engines, propellers, fatigue evidences, electricity, instruments, fuels, and a host of others.

The overhaul mechanic needs to know the same with a trend towards specialization.

The Radio Operator needs to know code, voice modulation, static, mechanical operations of receivers and transmitters.

The Radio-repair Mechanic needs to know transformers, con-

densers, coils, tubes, voltage, ampherage, resistance, frequencies, in fact all technical knowledge of a professional radio man.

The military forces have used radar very effectively. Air lines in the future will use radar and television. The Pilot will look on the instrument panel at his television receiver and see thru night, the storm or fog, all that he could see in the clear day time. The personnel for this equipment will have to be trained.

Practical Application of Knowledge

To possess knowledge is one thing. To know how to use or apply the knowledge is often another thing. Industry wants a student, who takes a physics course to learn how to apply the laws he learns.

Johnnie of the ferry boat started learning physics when he was seven. He learned it even thru the help of a stranger. He also learned in those childhood days how to apply that which he learned. Sometimes he learned how to apply the law before he learned the "why" of the law.

May I illustrate further from a personal experience? I was reared in a small community, where practically each home had a small farm. I learned to use a "pry-pole" to move heavy things. My physics teacher stated, "Now we are to study the fulcrum. If one of you could get a fulcrum long enough, and would place it over a fulcrum point, if you could find one, you could lift up the earth." I thought to myself, "Fulcrum must mean a pry-pole. The fulcrum point must mean the box I used, which broke when the object was too heavy." I said to myself, "Fooey, how are you going to find a fulcrum point strong enough to lift the weight of the earth." I lost interest in the teacher and his lesson on fulcrums.

If my physics teacher had made four demonstrations of the use of a fulcrum, it is believed that every student would have been interested. Demonstration one, the successful use of a fulcrum. Number two, the unsuccessful use of a fulcrum wherein the fulcrum was not strong enough therefore would break. Number three, the unsuccessful use of a fulcrum point wherein the point was not strong enough therefore it would give way. Number four, a repeat of a successful use of a fulcrum.

Begin with a positive and end with a positive. We all love success. To many, the beginning impression is very strong and to others the final impression is strongest. But all of us like a successful endeavor.

Industry therefore expects a student to get from a physics course, three things:

First: a wholesome mental attitude towards life, as lived with nature, with a single human being, and towards social groups.

Second: to find an interest in and an aptitude for some field of endeavor.

Third: an all-round knowledge of the fundamental laws of basic high school physics, which can be applied in the work-aday-life.

THE QUIZ SECTION

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1. An equation of degree n has exactly n roots. (T or F)
2. Oxidation is the *loss* of electrons by the atom or substance oxidized. (T or F)
3. At 0° C. ice is lighter than water and floats. (T or F)
4. A wheel rolling from left to right *along the ceiling* turns counter-clockwise. (T or F)
5. In ethane, C_2H_6, the valence of carbon is 3. (T or F)
6. There are no real logarithms of negative numbers. (T or F)
7. What is the probability of selecting an ace from the usual deck of 52 cards?
8. How long does it take a sum of money to double itself at 6% interest compounded annually?
9. Write 213_6 in base ten.
10. $Y = \sin 3x$ has a periodicity of 120°. (T or F)
11. It is impossible to form a real image of a real object on a screen if the distance from object to screen is less than $4f$ (T or F).
12. What is a decibel?

ANSWERS TO THE QUIZ SECTION

1. T; 2. T; 3. T; 4. T; 5. F; 6. T; 7. 1/13; 8. 11.9 years; 9. 81_{10}; 10. T; 11. T; 12. Approximately the smallest *change* of loudness that the ear can detect.

AIR EXPRESS BLANKETS U. S.

With domestic airway routes recently mounting to 67,000 miles, the Air Express Division of the Railway Express Agency now estimates that its combined transportation facilities cover almost 300,000 miles in this country.

Adding to the increased air routes, the 200,000 miles of railway, 12,000 miles of waterways and 16,000 miles over America's roads, a coordinated system of 295,000 miles of transportation blankets the U. S.

The Agency also acts as agent for airlines flying air express to South America, Europe and the Orient.

ELEMENTARY SCIENCE

SCIENCE ELEMENTARY CURRICULUM COMMITTEE
Hammond, Indiana

THE PURPOSE OF TEACHING ELEMENTARY SCIENCE

The fundamental purpose of teaching Elementary Science is to initiate the child into the natural environment in which he lives,—the world and universe at large,—and to establish desirable attitudes and re-actions. Subject matter is not to be thrust upon the pupil. It should grow out of experience. The teacher's part is to create in the pupil a desire to learn.

John Burroughs struck the proper key-note for the teaching of Elementary Science when he said

"I should not try directly to teach young people to love nature so much as I should aim to bring nature and them together and let an intimacy and understanding spring up between them."

BASIC OBJECTIVES IN TEACHING ELEMENTARY SCIENCE

1. To give the child an understanding and appreciation of his immediate environment.
   a. Natural World—Plants and Animals
   b. Physical World—Earth and Universe
   c. Forces—Wind, Fire, Electricity, Sun
   d. Phenomena—Rainbow, Northern Lights, Rock Formation
   e. Changes—Weather, Time, Seasons

2. To lead the child in the development of scientific attitudes.
   a. Questioning attitude toward things in his environment.
      Interest in
      Curiosity about
   b. Open Mindedness
   c. Basing Conclusions upon facts

3. To develop social attitudes and appreciations which have a beneficial effect on behavior.
   a. Recognition of the fact that we should "Use but not abuse." Abuse and waste can lead to destruction, e.g.—balance of nature, natural resources, human life and governments.
   b. Understanding of the inter-relationship and interdependence between plants, animals, and man.

4. To teach appreciation of the beautiful and foster reverence toward the Creators handiwork.
   a. The organization and plan of nature

5. To train the child in the scientific method of thinking.
   a. Recognize a problem
   b. Gather information—observation, experimentation, reading, drawing, on own experiences.
   c. Record information

d. Evaluation of facts
e. Arriving at a conclusion

6. To provide the necessary information which will contribute to the health, safety, comfort and convenience of the individual.

7. To make practical use in daily living of the things found out from scientific thinking.

8. To open up new avenues of interest and satisfaction.

The Organization of Materials of Instruction

During the course of the past school year a questionnaire relative to science was given each teacher in the elementary grades. The science committee devoted considerable time to the study of the answers in this questionnaire. It was decided that the Science committee would sub-divide itself into smaller groups; each group to devote its time to a specific grade. Also, each group was to work out a specific unit for its grade level. The topics given preference in the questionnaire, are those which have been chosen by the sub-committees for their units.

Kindergarten—A complete scope of work has been developed at this level.
First Grade—Unit on "Water."
Second Grade—Unit on "Weather."
Third Grade—Unit on "The Solar System."
Fourth Grade—Unit on "The Earth and Life Upon It."
Fifth Grade—Unit on "Changes in the Weather."
Sixth Grade—Unit on "Sound."

The plan followed in the organization of each unit is somewhat similar to that followed in Cleveland, Ohio. Following are the general headings which we used: *Purpose—Things to Learn —Things to Do—Understandings—Bibliography.*

There is also a space for the comments of the teacher. This is of great importance. This committee wants the comments of each and every classroom teacher relative to the material presented. Suggestions for added material or criticism of the material presented will be appreciated.

Basic Text Books were analyzed from each of the following publishers: Scott Foresman and Co., The L. W. Singer Co., Ginn and Co., and Allyn & Bacon. Each of the above companies has a series of elementary texts for science. At this time, the committee did not want to confine itself to one specific text. As a result, the books used were chosen, for the most part, from the above four publishers.

The members of the Science Committee feel that this is only an experimental beginning in the field of elementary science. With the help of each classroom teacher a scope of work in the field of elementary science can be developed eventually that will be practical and usable for the Hammond schools.

Shortly after the opening of the schools in September the teachers at each grade level will meet with the corresponding science committee for a discussion of the material developed by the committee.

SOUND

A Sixth Grade Unit

I. Purpose.
To bring to the attention of the pupils the science of sound and its control.
To emphasize the delicate nature of the mechanism of the ear and the consequent necessity of care of the ear.

II. Function of the Unit.
The unit presents a simple, well rounded treatment of the principles underlying sound. To really learn a principle a child must practice using the principles as a method of explanation in all sorts of situations. Therefore, many experiments are suggested so each child has a chance to observe, experiment, and draw conclusions. It is also hoped the teacher will capitalize on the many opportunities that afford careful experimentation and the drawing of conclusions only after sufficient evidence has been acquired to justify the conclusions. This will show children that it requires more than one demonstration or experiment to bring about an understanding.
Children should be encouraged to devise many experiments of their own. This unit offers many opportunities for children to have real fun. To facilitate a wide use of materials it is suggested the class be divided into groups to perform experiments after the problem is introduced and set.
It is further suggested that each teacher perform those experiments throughout the whole unit herself before she teaches the class.

III. The following is a list of equipment needed for this unit.

Ruler
Rubber bands
Metal triangle
Tuning-fork
Pan or dish
Small cork
Thread
Blades of Grass or small strips of cellophane or paper
Two tuning-forks exactly alike.
Pail
Yardstick
Strip of cloth
Cardboard or megaphone
Chalk box
Three rubber bands, same length but of different widths
Three rubber bands exactly alike
Spools on which they can be stretched
Xylophone
Violin
Flute
Saxophone
Trombone
8 test-tubes same size
Test-tube rack

Things to Learn	*Things to Do*	*Understandings*	*Comments*
I How are sounds made?	Read Manuals of Discovering World III. pp. 52–56. How and Why Discovery. pp. 165–166. Observe different kinds of sound makers. a. Ruler b. Rubber bands c. Triangle d. Tuning-fork Observe that the prongs of a tuning fork vibrate when they are struck.	All sound is made by vibration.	
II How do living things make sound?	Listen to sounds made by living things. a. Birds b. Frogs c. Dogs d. Insects e. Cats Find out whether there is movement in the throat when sound is coming from it. a. Hum a tune b. Hold your throat tightly enough to feel the quiver.	All sound is made by vibration.	
III How does sound travel?	Show how moving air causes vibration. a. Find blades of grass. Hold them between your thumbs and blow on them. b. Hold pieces of cellophane or paper tightly between your thumbs and blow on them. Show that sound waves travel through the air. a. Use two tuning-forks that are alike. Set one tuning-fork vibrating and then hold the other fork very close to it. Do not let the forks touch each other. Observe that the second fork vibrates. Show that sound travels in every direction. a. Have someone ring a bell. Walk around the bell as it is ringing. Show that sound travels in solids, liquids and gases. a. Use a yardstick and a tuning-fork. (See DOW III, pp. 120) b. How and Why Discoveries. pp. 46 Show that sound waves can be guided. (page 122 DOW)	Sound travels by vibrations in the air, or sound waves.	
IV What makes the differences in sound?	Find out why some sounds are high and others are low. pp. 125–126.	The pitch of a sound depends upon	

Things to Learn	*Things to Do*	*Understandings*	*Comments*
	DOW III. How and Why Club, pp. 58–61.	the speed of vibration.	
	Show that loud sounds and soft sounds came from the same source. (DOW III, pp. 128)	The loudness of a sound depends on how hard the object is vibrated.	
	Show that vibrations may be great or small. (page 129) (DOW III)		
V How do musical instruments make sounds?	Show how different sounds are made by musical instruments. (pp. 132–137 DOW III.) How and Why Discoveries, pp. 61–69	All musical instruments produce sounds by making the air vibrate.	
VI How do we hear sounds?	See how the ear helps to catch sound waves. (DOW III 139–140) (Sound, pp. 28–34)	We hear sound because the sound waves travel to the ear and make its inner mechanism vibrate.	
	Show that one can tell from which direction the sound is coming. (DOW II, p. 140.) SOUND, pp. 28–34.		

Bibliography

1. *Children's References*

Beauchamp, W. L.; Mebrose, Mary; Blough, Glenn O.: *Discovering our World*, III, pp. 109–143. Scott Foresman and Company, Chicago, Illinois.

Frazier, G. W., et al: *How and Why Discoveries*, pp. 46–69. L. W. Singer and Co., Syracuse, N. Y.

Carpenter, Harry A. et al: *Adventures in Science with Ruth and Jim*, pp. 123–140. Allyn and Bacon Publishing Co.

Parker, Bertha; Blough, Glenn: *Sound*. Row Peterson and Co. Chicago, Illinois.

2. *Teacher's References same as above, also:*

Manuals for the following:

Discovering our World III, Scott Foresman and Co. Chicago, Illinois.

Teacher's all Science Handbook, L. W. Singer and Co., Syracuse, N. Y

I think by far the most important bill in our whole code is that for the diffusion of knowledge among the people. No other sure foundation can be devised, for the preservation of freedom and happiness. . . . Preach, my dear sir, a crusade against ignorance; establish and improve the law for educating the common people. Let our countrymen know that the people alone can protect us against these evils, and that the tax which will be paid for the purpose is not more than the thousandth part of what will be paid to kings, priests, and nobles who will rise up among us if we leave the people in ignorance.—*Thomas Jefferson to George Wythe, Aug. 13, 1786.*

FABRICATION OF ELECTRICAL DIAGRAMS

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INTRODUCTION

Drafting of electrical diagrams has long been a time-consuming, and hence costly, process. Although any reasonably competent engineering draftsman can draw intelligible electrical diagrams, only a few can produce really good diagrams, and their production is very small, leading to high cost production.

Simplification of many forms of drafting, by use of lettering guides, stencils, Ben Day applications and "pasteup" techniques, has been progressing for the last two decades.[1] Because most electrical symbols are too complex for standard lettering guides and stencils, or for production by simple drafting machines,[2] simplification of electrical drafting has lagged behind that in other fields.

OUTLINE OF PROCESS

Fabrication of electrical diagrams consists of cementing standard stock symbols or symbol assemblies in place on a rough guide sketch, drawing in the necessary connections, photographing the whole, retouching the copy negative to remove unwanted "joint lines," and printing the finished diagram from the retouched copy negative.

By use of this process, the original can be made in any convenient large size, and the finished print can be of any convenient small size (within reason). Any number of copies can be printed from the copy negative.

Fabricated diagrams cost slightly more for materials than those drawn entire, but the labor involved is considerably less, and the finished job usually is of considerably better quality, so that the over-all cost—materials plus labor—is lower.

ELECTRICAL SYMBOLS

Electrical symbols in use today are made according to three systems. Radio diagrams employ one system (as do most physics

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[1] See reports of U. S. Department of Agriculture and U. S. Geological Survey since 1930 for development of processes; also Ives, R. L. *Fabricated Diagrams*, Journal of Geology, 47, 1939, 517–545.

[2] Examples: WRICO and LEROY lettering guides; also Thornthwaite, C. W. and Stuart-Sharpe, C. F. *Patterns on Maps and Diagrams by Carbon Transfer Process*, Science, 91, 1940, 367–368.

and engineering texts); industrial diagrams are drawn according to a somewhat different system, although the symbols are not all different; and a third system, which attempts a compromise between the radio and industrial systems, has recently been worked out by the A.I.E.E.

None of these systems have standard symbols for all pieces of electrical equipment, so that each manufacturer has adopted a set of special symbols to represent items such as stepping switches, ratchet relays, thermal time delay assemblies, etc.

Symbols representing familiar simple electrical components are shown in Fig. 1. These, it should be obvious, take considerable time to draw, and are difficult to draw neatly in small size.

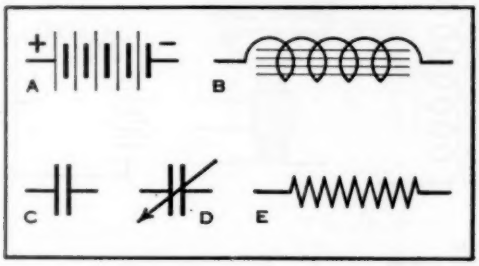

FIG. 1. Simple electrical symbols. *A*—battery; *B*—resistor; *C*—fixed condenser; *D*—variable condenser; and *E*—iron-cored inductance. Addition of an arrow diagonally across any symbol indicates that its value is manually variable.

These symbols, however, can be drawn neatly and quickly in a large size ("half as big as the drawing board"), and then reduced photographically to a convenient working size, such as twice final size, and any number of prints made, on suitable paper, for later use.

SYMBOL ASSEMBLIES

In many instances, where an assembly of symbols is used repeatedly, it is found desirable to make a master drawing of the entire assembly, and from it working prints, in convenient size, for use in diagrams. Assemblies of more complex symbols are shown in Fig. 2.

More complicated electromechanical devices, such as latching relays and stepping switches, can likewise be drawn as single diagram components in large size, and then reduced to working size. The saving in drafting time, when this is done, is enormous, as can be judged from reference to Fig. 3.

When a standard symbol, either simple or complex, is drawn, it is usually found economical to include all parts of the device. Those not wanted in a given drawing can be eliminated by use of scissors or an etching knife. As the cost of a print is determined by its area, rather than by the number of lines on it, this procedure is economical. As a specific example, should a simple (nonlatching) relay with a 3 pole, single-throw, normally-open contact stack be desired, it can be made from the relay assembly (Fig. 3, lower left) by clipping off the latching coil and the upper four contacts of the "stack," and etching out the top

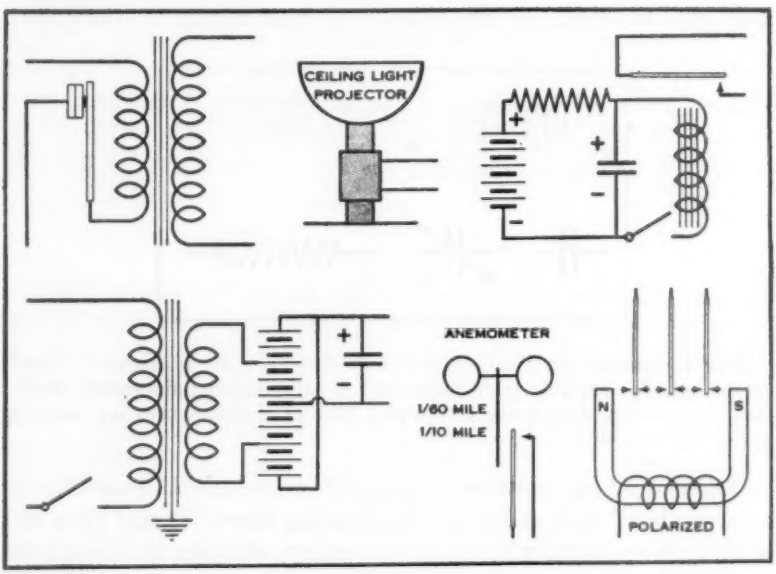

FIG. 2. Symbol assemblies. Top—microphone hummer (left); ceiling light projector (center); relay accelerator (right). Bottom—rectifier and filter (left); anemometer (right).

contacts on the remaining three contact arm assemblies. This takes approximately one minute per relay. Similar eliminations of unwanted parts can be made from other components.

DIAGRAM CONSTRUCTION

Diagram construction is relatively easy, and is accomplished by the following steps:

1. Lay out base sketch of diagram, to approximate scale. It may be desirable to use component prints to determine arrange-

ment and spacing. Allow room between components for necessary line work.

2. Cement component prints to base sketch with any good grade of rubber cement. Line up components with a T square. Make sure edges are firmly stuck down.

3. Allow rubber cement to dry hard, then remove excess cement from around and on prints. Ink work will not "take" on paper with rubber cement on it.

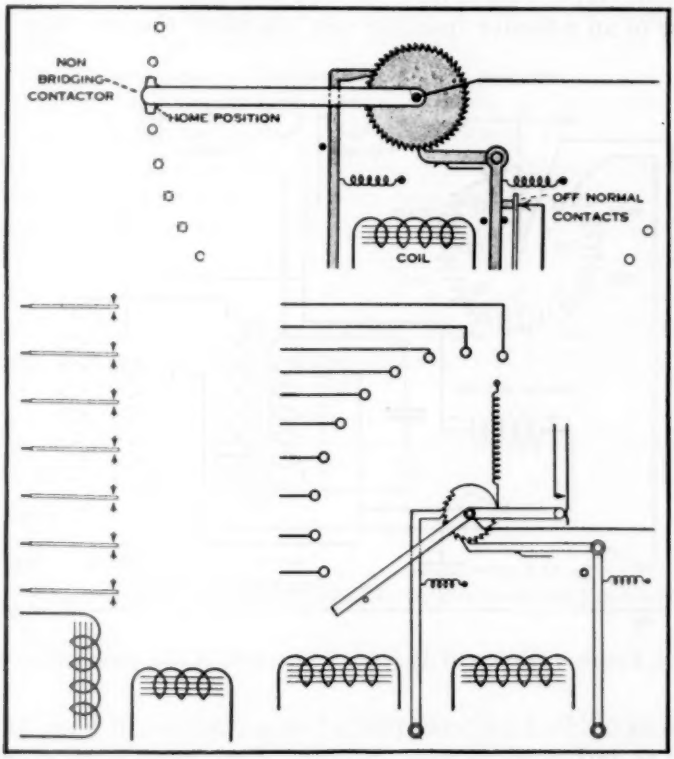

FIG. 3. Master assemblies of complex electromechanical devices. Top—part of a continuous rotation stepping switch; bottom left—latching relay with multi-contact "stack"; bottom right—spring return stepping switch.

4. Draw connections in pencil, then ink over them.
5. Cement on necessary captions.
6. Check diagram for accuracy, omissions, and neatness.

The diagram is now ready for copying. A finished diagram made by this method comprises Fig. 4.

Use of Ben Day

Where interrupted lines are necessary, as to indicate a virtual component, such as the distributed capacity of a coil, line work may be simplified by superposing negative Ben Day over a solid line.

Ben Day (named for its discoverer) is a transparent sheet with line patterns printed on it. This is supplied in many forms, the most convenient for the electrical draftsman being the self-adhesive type, which, once pressed into place, stays there because of an adhesive (paraffin wax) backing. Positive Ben Day

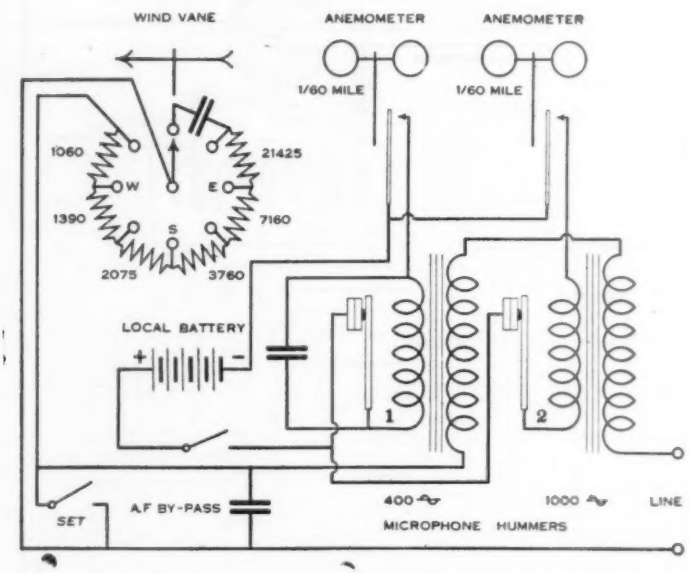

Fig. 4. Finished fabricated diagram. The circuit is of a wind transmitter.

consists of black patterns printed on a transparent base, and is used to darken white areas. Negative Ben Day consists of white lines printed on a transparent base, and is used to lighten black areas.[3]

Conversion of a solid line to an interrupted line by use of negative Ben Day is shown in Fig. 5 (left). Use of stipple positive Ben Day to fill a white area is shown at the right of the same figure. This use is to indicate a gas-filled tube. Without it, the tube is considered to be of the high vacuum type.

[3] Made by Para-Tone Mfg. Co. of Chicago and Craftint Mfg. Co. of Cleveland.

Lettering

Lettering of electrical diagrams can be done freehand, as is customary; by use of lettering guides, as is becoming more common; or by cementing into place the desired printed words or symbols.

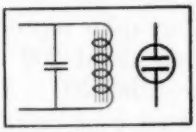

FIG. 5. Use of Ben Day. Left—interruption of line by use of negative Ben Day; right—darkening of white area by use of positive Ben Day.

Use of printed words and symbols would, at first consideration, seem extremely costly. Actually, because the vocabulary used on electrical diagrams is extremely small (about 60 words), costs over a period of time are very reasonable.

Type for use in fabricated diagrams should be chosen with care, bearing in mind that it must stand considerable reduction. This requires that simple faces, such as "Copperplate Gothic" be used (as in Fig. 4). As cost of printing is almost all in the typesetting, a large number of impressions should be run off at one time.

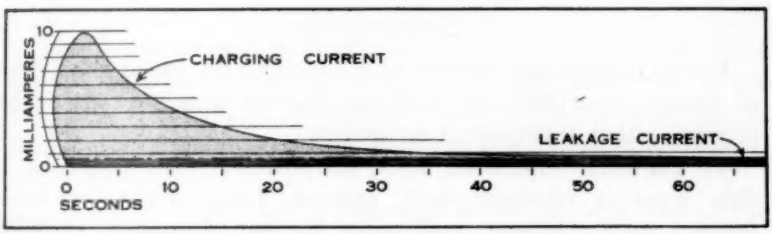

FIG. 6. Diagram using type cemented in place. This procedure, in general, is far superior to either freehand or guided lettering when a large number of figures are to be made.

Printed words should be arranged with wide spacing between them, so that they can be cut out of the type card easily, and with a reasonable white margin around the printing. Costs will be reduced by proper choice of words. For example, if the word DISCONNECTORS is printed, it can be trimmed to DISCONNECT, CONNECTORS, CONNECTOR, CONNECT.

Size of printed words is determined by multiplying the size desired in the finished figure by the copying reduction ratio, and

choosing the nearest stock type face to this figure. Special type faces are extremely expensive.

Should special symbols be needed in large numbers, they can be drawn to any convenient large size, then a line cut made to desired dimensions, and any reasonable number run off by the printer. In general, photographic reproduction is most economical for numbers less than about 200 (all made at once); and printing for numbers exceeding 200 in a single order.

A diagram using printed type cemented in place comprises Fig. 6. Lettering in Fig. 4 was similarly inserted.

Copying

Completed fabricated diagrams may be copied by a number of photographic methods. All that is needed is a reasonably good camera and a contrasty film. "All or nothing" films, such as Kodalith and Reprolith, are preferable for this work, as are contrasty developers.

In most instances, it will be found economical to have copies made by a commercial photographer or planographer, whose equipment is arranged for rapid and inexpensive copying of this type. Most planographers do all necessary spotting of copy negatives as a part of the copying process.

Retouching

Retouching of copy negatives is considerably easier than that of scenic or portrait negatives, as all areas on a line copy negative are either transparent or opaque.

To eliminate undesired lines, fill pinholes, etc., paint on a thin layer of photographers' opaque. Large areas are best treated with a brush; small areas by use of opaque, slightly diluted, in a ruling or lettering pen.

Lines can be inserted in a negative by engraving with a photographers' etching knife, or jewelers' graver.

It will be noted that all copyable defects in an original will necessitate work on the copy negative. In consequence, sloppy work on the original will require an inordinate amount of retouching, which is more time-consuming than correction of defects on the original.[4]

Printing

Making of prints from copy negatives is a standard photo-

[4] Ridgway, John L. *Scientific Illustration*, Stanford University, 1938.

graphic process, differing from other work only in that a more contrasty final result is desired. This is best achieved by use of a good negative, a medium-contrast paper, fresh developer, and considerable care in printing.

Overprinting or overdeveloping will cause spreading of lines over the entire print. Poor contact between paper and negative will cause a localized spreading of lines. Underprinting or underdeveloping will make the lines weak or "washed out" appearing.

Printing papers most suited for this work are glossy papers, properly ferrotyped; or semi-matte papers, such as "Adtype." For binding in reports and theses, "Adtype" is somewhat superior, as it will not crack on ordinary bending, as does glossy paper.

Print Retouching

Although all errors and defects should be removed from the copy negative, they may be removed from the print by use of an etching knife. Omissions from the original may be drawn into the print by use of a suitable pen, provided the paper surface is clean and grease-free. It should be noted, however, that any defect not corrected in the original, or on the copy negative, must be corrected in each and every print made from the negative.

Workmanship

Many defects not passable on a line drawing can be disregarded on a fabricated diagram, as they will "drop out" in the copying. It should be emphasized, however, that use of "paste-up" methods is no excuse for sloppy or careless workmanship at any stage.

Defects not corrected in the original can be corrected on the copy negative, it is true, and those not eliminated from the copy negative can be fixed in the final print, but, as retouching is more costly and time-consuming than drawing; and print retouching requires as many corrections as there are prints, it is essential that each defect be corrected in the earliest stage possible.

Conclusions

Practical experience with the fabrication of electrical diagrams indicates that it is a useful process by which a draftsman of ordinary ability can produce better-than-average electrical diagrams at less-than-average cost.

Where a large number of copies of a diagram are to be made,

such as for a thesis or report, fabricated diagrams are probably superior to those made by any other method. The finished print from the copy negative is in standard form for use by the planographer or engraver, needing no "patching" or "fixing up" before he begins work.

Cost of making fabricated diagrams is considerably lower than that of "straight" drafting, and the quality of the finished figures, in practically every instance, is very much better.

ACKNOWLEDGEMENTS

The writer is indebted to the Colorado Blueprint Co., of Denver, for much assistance with the development of these methods of making diagrams; and to T/3 Evelyn L. Kroona, WAC, for secretarial assistance with the preparation of this paper.

CENTRAL ISSUES IN THE PROGRAM OF MATHEMATICS FOR A WORLD AT PEACE*

WILLIAM BETZ

Chapman College, Los Angeles 27, California

Dr. William Betz, specialist in mathematics, Rochester, New York, and a member of the Commission on Post-War Plans appointed by the National Council of Teachers of Mathematics, spoke on *Central Issues in the Program of Mathematics for a World at Peace.*

The speaker outlined briefly the notorious hostility toward mathematics on the part of our educational leaders, during the two decades preceding the recent world war. Thus, in the majority of our states not a single hour of mathematics was required for graduation from high school. A moderate amount of "grocery store" arithmetic was declared to be sufficient for the needs of all but a few "specialists." The war brought a rude awakening. The Armed Forces soon discovered a shocking state of unpreparedness, not only in secondary and higher mathematics, but even in the rudiments of arithmetic. Thus, Selective Tests showed that only one inductee in three was able to select the correct answer from four suggested answers for "$7-5\frac{3}{4}$". A heroic training program became necessary, at terrific expense

* Abstract of an address before the Mathematics Section of the Annual Convention of the Central Association of Science and Mathematics Teachers, November 23, 1945.

to the nation, to remedy the worst features of this lamentable ignorance.

What should be our attitude toward this situation, and what is to be our orientation concerning this problem now that peace has returned to the world?

The war has revealed to millions of our young people that both in war and in industry mathematics is our sharpest weapon of national defense. The end of the war was hastened by the atomic bomb, the joint contribution of mathematics and science. That fact alone, together with the coming industrialization of Russia and China, and the revolutionary readjustments demanded by a global economy in the air age we are entering, would make a continuance of our neglect of mathematics synonymous with national suicide.

In February, 1944, the Commission mentioned above was organized for the purpose of studying the problem of mathematical instruction in the post-war world. After a careful study, the Commission published two reports which outline its recommendations.

The substance of these reports may be summarized as follows: We need a continuous, integrated program in mathematics extending from the primary grades through the junior college years. A check list of basic mathematical objectives, constituting a definition of literacy in elementary mathematics, has been prepared. The teaching of arithmetic must be completely reorganized. It must be made systematic, meaningful, and thorough. The junior high school period must stress certain essentials on a nation-wide front. In the high school we need a two-track program. In addition to the traditional courses we must have a two-year course in general or basic mathematics which gives more attention to the "functional" aspects of mathematics. In all courses we must eliminate non-essentials and stress meaning, understanding, and mastery. A good deal of emphasis should be put on the place of mathematics in the modern world. The experience of the Armed Forces in developing training films, film strips, and other teaching aids, should be utilized as far as possible. Above all, we need teachers whose training and outlook can cope with these new demands.

In conclusion, the speaker asserted his personal belief that the possibility of effecting this necessary reorganization will ultimately rest on a prior redirection of our educational philosophies and our administrative policies.

JUNIOR HIGH SCHOOL MATHEMATICS

WHY IS THE ABILITY TO RECOGNIZE RELATIONSHIPS IN READING OF PROBLEMS SO VITAL, AND WHAT METHODS MAY BE USED TO DEVELOP THIS ABILITY?*

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Supervisor Elementary Instruction, Lake County, Crown Point, Indiana

Civilization and mathematics have had a parallel development. Mathematics grew out of the needs of man and at all stages of his progress it has been his aid. It has even been stated that there have been times "when civilization could not advance and stopped temporarily waiting for further development in mathematics." Certainly today mathematics fills a greater need than heretofore. Everyone has experiences involving number. Modern industry, business, and science are thoroughly dependent upon the subject. Government agencies, people in all professions and in all phases of home life use quantitative problems which arise in dealing with budgets, bills, accounts, purchases, travels, parcel post, installment buying, saving, borrowing, investments, pensions, insurances, annuities, and income taxes. The value of mathematical knowledge has been emphasized in war by the military authorities. Our schools have received criticisms for failing to develop accuracy and speed in the fundamentals and the ability to solve problems.

The knowledge of algebra is becoming an essential today to an increasingly large number of people in scientific investigations, in research, in designing, and in operations.

The ability to comprehend very large numbers and graphic ways of expressing numerical facts is necessary. Our newspapers carry such items daily.

The methods of thinking used over and over in solving mathematical problems are not unlike those used in solving the problems of everyday life in all fields of activities. Hence the technique developed through constant practice in mathematics can be applied to the problems which occur in our daily activities. This technique involves an analysis of conditions. Concepts must be clear. The known facts must be identified and kept in mind. The relationships involved in the problem

* Presented to the Junior High School Group of the Central Association of Science and Mathematics Teachers, November 24, 1945.

must be understood. The facts to be found must be listed. A choice must be made of the most effective procedure leading to the solution. Thinking of this type is necessary in all activities in which problems arise—political, social, industrial or commercial.

Because of the importance of the subject and the difficulties involved, the teaching of problem solving in arithmetic offers one of the greatest challenges to Junior High School teachers.

The ultimate purpose in arithmetic teaching is to develop the ability to think in quantitative terms. A criticism directed to all levels is that children do not think. With proper attention to the complexity of the subject, the pace of learning, and the emphasis upon relationships, which has been indicated, there should be improvement in the ability to think.

The place of the teaching of relationships in the total arithmetic program having been pointed out, the next step will be to consider methods of teaching them and the relation of reading to problem solving.

Each new step should be definitely related to preceding steps. Help the pupil to visualize the parts so he can see the number relationship involved. Oral problems of real or imagined situations should precede reading problems which are to give further understanding and experience.

The ability to comprehend the printed page is necessary in any subject therefore the mathematics teacher cannot escape being a teacher of reading also. The pupil, especially during his early experience in grade seven will need much help if he is to make a satisfactory adjustment to his new environment. Reading probably needs more attention than any one factor if we bridge the gap between the elementary and Junior high school levels. On a recent visit to a ninth grade algebra class I found that this problem gave almost all the pupils difficulty: The sum of two consecutive numbers is 17. What are the numbers? The word *consecutive* had no meaning for most of them. Some had not looked it up. Others had but didn't get the correct meaning. They said: "It means to follow each other." Only one in the entire group gave a clear definition.

Most textbooks have made great improvement in their vocabularies and sentence structure but there are many words peculiar to the subject that pupils must understand before the problem has meaning.

It is necessary that teachers be aware of the types of diffi-

culties children meet in their mathematical readings. One of the best studies is the one made by Dr. J. S. Georges. In his study, vocabulary difficulties headed the list with 23.4 percent. Other reading difficulties were classified under lack of: interpreting illustrative materials, intensity in reading, ability to analyze, precise reading or manner of statement of exercise.

Some studies have been made to determine the relation between vocabulary and problem solving. Some report a definite correlation, others slight.

In 1938 it was found that the pupils of the Lake County elementary schools ranked below the national norm in both problem solving and vocabulary. The most alarming deflection below the norm was in the literature test as measured by the Stanford Achievement Battery. Greater library facilities were provided on classroom reading tables, in the school libraries, extension centers and Book Trailers provided by the Gary, Crown Point, Lowell, and Hebron libraries. Test results in recent months showed not only that literature was up to standard but that the vocabulary and problem solving results were even higher. Directed attention to these difficulties raised the scores on all three but most in problem solving. From research and experience it seems clear that improvement in reading skills results in improvement in problem solving.

Reading Mathematics Problems. Principles to be Observed

1. The reading of problems in mathematics is a technique involving recognition of what a given phrasing may connote or imply beyond what it specifically states.
2. There will never come a time when the mathematics teacher can take for granted the ability of pupils to read mathematics with understanding until they have had special attention given to the reading techniques employed in efficient mathematical study.
3. Mastery, then, of the *vocabulary* and *phrasing* applicable to mathematics will be involved in good reading of mathematics.
4. The specialized vocabulary of mathematics is very extensive.
5. Mathematics employs a symbolic language and is often a difficult process to translate ordinary word statements into this symbolic language.

6. Mathematics is read for exactness of comprehension and mastery not for appreciation and general ideas.

7. Every help that the teacher can give in reading mathematics will yield results.

Methods of Improving the Reading of Problems

1. Readiness for problem solving should be stimulated by discussion of the situation, by relating it to incidental quantitative situations, and by a demonstration showing the difference in techniques used in reading a paragraph in literature and a problem in arithmetic. One rapid reading may be sufficient for the first but a slow, careful, and repetitive reading for the second.

2. Presentation of the new step. Each new step is presented in a concrete problem situation which should be discussed with the pupils before the computational work is begun (each part studied in relation to the other). Pertinent pictures, illustrations, diagrams, and similar visual aids that are given should be studied so that the meaning of the computation is clear. Then the illustrative solution should be presented at the blackboard. Later this explanation will serve as a reference for the pupils who should review the step for some reason, such as failure to recall the process, or difficulty in a diagnostic test.

3. Problems should be significant—they should present a familiar social situation.

4. Have pupils read from their arithmetics. Drill in first reading—attention to conditions of the problem and identification of the process. In second reading note numbers and use care in copying them. Stress importance of reading with understanding.

5. Have them reproduce the problem, paying particular attention to pronunciation and vocabulary meaning.

6. Have pupils paraphrase the problem.

7. Have them state what the problem asks and the information given.

8. Diagram the problem.

$$\text{Money left} \begin{cases} \text{Total Amount} \begin{cases} \text{Amount at start } 16 \\ \text{Amount earned } 10 \end{cases} \\ \text{Amount spent } 15 \end{cases}$$

9. Practice many problems with different situations but requiring similar skills in thinking.

10. Formulate new problems.

11. Answer questions by teacher. Examples—

Does the problem tell you what the house cost?

Does the problem tell you how far it is from New York to Chicago?

What is the meaning of consecutive?

12. Pass judgment as to whether the facts are true to life situations.

13. Pupils may give any new information they have gained from the problem.

14. Encourage the reading of a large amount of subject matter in the field such as: a variety of texts, business sections of newspapers, the World Almanac.

15. Further reading such as:

How to use Pictorial Statistics
Mathematics for the Millions
Number, the Language of Science
From Immigrant to Inventor
The Queen of the Sciences

Conclusion

Morton: "The most important single factor contributing to the success or failure of the pupil is the teacher. It is imperative that the teacher know well the subject matter he is to teach; that he understand children, their interests and how they learn; that he have desirable personality traits; and that he know how to teach."

REFERENCES ON IMPROVING READING COMPETENCE IN MATHEMATICS*

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This bibliography represents the result of an effort to assemble a list of published references which would be helpful to teachers in their task of improving the ability of their students to read mathematical material. The effort has been, at best, only modestly successful. The whole array of books, articles, and research studies on the teaching of mathematics is noteworthy for its lack of concrete practical suggestions for achieving this end, rather than for any abundance of such suggestions.

* Presented to the Junior High School Group of the Central Association of Science and Mathematics Teachers, November 24, 1945.

There can be no doubt of the importance of proper reading habits as an element in successful work in mathematics. In view of this, it seems strange, and it is unfortunate, that there are very few studies or articles which go very far into detail in recommending specific ways of helping students attain reading competence in this field. There are several which do emphasize its importance as a conditioner of general mathematical success, but even these, for the most part, go no further. Even such a study as Symonds' "Special Disability in Algebra" contains no practical direct suggestions for improving the mathematical reading ability of the students.

This is not to say, of course, that there is no value in the available material on this subject. Quite the contrary is true. There are a few references which teachers will find immediately and specifically helpful. There are a good many others whose implications can point the way for teachers to work out their own remedial measures. In particular there is a group of writings on teaching verbal problems which indirectly suggest such measures. No attempt is made here to list all of these because they are so numerous, but any good book on the teaching of mathematics will contain references to a number of these. It seems a little strange that few of these references deal explicitly with reading *per se*, but the suggestions on problem analysis generally imply some specific ideas for putting more point and direction into the reading of the problems. Teachers will do well to consult some of these references.

In view of the varied character of the references available, it has been thought wise to make brief annotations of the ones included in the bibliography presented herewith.

In addition to this annotated list of references, the interested reader is referred to two other bibliographies (neither is annotated) of articles, studies, and books bearing on this general problem of reading mathematical material. These have been freely drawn upon in preparing the present list, but not all the references have been included here.

The first is part of an exhaustive volume, "A Bibliography of Mathematical Education," prepared by Professor William L. Schaaf of Brooklyn College and published in 1941 by The Stevinus Press, Forest Hills, New York.

The other is a bibliography of 44 references appended to the article "Remedial Reading in College Mathematics" by Professor A. L. O'Toole and included in the list presented herewith.

Many of the references listed in the following pages are to be found in various issues of two well-known journals: SCHOOL SCIENCE AND MATHEMATICS (the official journal of this Association) and *The Mathematics Teacher* (the official journal of the National Council of Teachers of Mathematics.) Since they are so well known to mathematics teachers the references to these periodicals are indicated, for the sake of brevity, merely by the initial letters *S.S.M.* and *M.T.*

AN ANNOTATED LIST OF REFERENCES BEARING ON THE PROBLEM OF IMPROVING THE ABILITY OF STUDENTS TO READ MATHEMATICAL MATERIAL

Barnet, Rudman: "Teaching the Verbal Problem in Intermediate Algebra." *M. T.*, Vol. 22, Feb. 1929, pp. 83–92.

Suggestions on the use of the analytic method, with some specific illustrations. In part these imply useful hints on helping students know what to look for in reading verbal problems.

Bennett, A. A.: "Algebra as a Language." *M. T.*, Vol. 30, Nov. 1937, pp. 307–313.

A thoughtful article by a noted mathematician who offers helpful suggestions to teachers for helping their students understand better the language of algebra.

Bertolli, J.: "The Mathematics Vocabulary of Current Periodical Literature." *M. T.*, Vol. 34, Nov. 1941, pp. 317–319.

Results of a word analysis of certain periodical articles for mathematical terms. The resulting vocabulary list is presented with some comments.

Boyer, Lee Emerson: AN INTRODUCTION TO MATHEMATICS FOR TEACHERS. New York; Henry Holt & Co., 1945, pp. 252–253.

Specific suggestions for helping students to improve their reading of mathematical writing. Teachers will find these suggestions pointed and helpful.

Breslich, E. R.: PROBLEMS IN TEACHING SECONDARY SCHOOL MATHEMATICS. Chicago; University of Chicago Press, 1931.

p. 23: on reading difficulties in mathematics.
p. 188: on the importance of correct reading habits in mathematics.
p. 276: suggestions for teaching students how to study a proof.

Breslich, E. R.: THE TEACHING OF MATHEMATICS IN SECONDARY SCHOOLS (TECHNIQUE). Chicago; University of Chicago Press, 1930, pp. 102–105.

Suggestions for training students to develop good reading habits in reading mathematical material.

Brooks, S. S.: "A Study of the Technical and Semi-technical Vocabulary of Arithmetic." Educational Research Bulletin (Ohio State University), Vol. 5, May 26, 1926, pp. 219–222.

A good pointed discussion of the problem of reading in arithmetic, with a vocabulary list of technical and semi-technical terms rated according to frequency of occurrence. The author's comments and suggestions are well worth the careful attention of teachers of arithmetic.

Brueckner, Leo J.: DIAGNOSTIC AND REMEDIAL TEACHING IN ARITHMETIC. Philadelphia; John C. Winston Co., 1930, pp. 312–217.

Exercises in careful reading of problems in arithmetic.

Buckingham, G. E.: "Relationship Between Silent Reading Ability and First Year Algebra." *M. T.*, Vol. 30, March 1937, pp. 130–132.

Technical. Emphasizes the need of improving ability to read mathematical writing, but does not tell how it can be done.

Carnahan, Walter H.: "Alternatives, Conventions, and Exceptions in Mathematics." *M. T.*, Vol. 35, Nov. 1942, pp. 310–312.

Attention is drawn to certain typical trouble spots due to faulty reading, or uncertain interpretation of mathematical statements.

Cowley, E.: "Technical Vocabularies for Plane and Solid Geometry." *Jour. of Ed. Research.*, Vol. 27, Jan. 1934, pp. 344–354.

This article extends an earlier report on the same subject by the same author. It summarizes some previous studies and presents results of some original work on technical vocabularies of school geometry. Part IV of this article contains some distinctly helpful suggestions that could be of use to the classroom teacher.

Drake, Richard: "Vocabulary Instruction in Mathematics." *M. T.*, Vol. 32, Apr. 1939, pp. 166–168.

Brief and somewhat technical. Emphasizes the importance of improving the reading ability in algebra but does not tell how.

Duncan, M.: "Meaningful symbols As a Better Preparation for Junior and Senior High School Mathematics." *M. T.*, Vol. 32, Apr. 1939, pp. 169–171.

A plea for more emphasis on the meaning of mathematical terms and symbols, especially those of arithmetic, in the early grades.

Durell, F.: "Making the Concrete and the Abstract Help Each Other in Mathematics." *S.S.M.*, Vol. 29, Oct. 1929, pp. 702–713.

An interesting discussion with some illustrative descriptions of methods used in clarifying mathematical concepts. It thus has a bearing on the problem of reading in mathematics.

Edwards, A. S.: "A Mathematics Vocabulary Test and Some Results of an Examination of University Freshmen." *Jour. of Ed. Psychology*, Vol. 27, Dec. 1936, pp. 694–697.

" . . . It is doubtful if some of the students are able to read the mathematics texts with proper understanding because of lack of knowledge of the terms used . . . ."

Description of an experiment. The technique could be adapted by teachers for similar use in the upper grades and secondary school. An interesting account. Brief and readable.

Georges, J.: "Nature of Difficulties Encountered in Reading Mathematics." *School Review*, Vol. 37, March 1929, pp. 217–226.

One of the best articles on the subject that has appeared anywhere. Reading difficulties are analyzed into six types, and suggestions are given for detecting and overcoming these difficulties.

Georges, J.: "On the Nature of Algebraic Language." *M. T.*, Vol. 21, March 1928, pp. 135–150.

Suggestions designed to help the teacher help the student in interpreting the symbolic language of algebra.

Georges, J.: "Training Teachers for Relational Thinking." *M. T.*, Vol. 34, Jan. 1941, pp. 3–7.

The author calls attention to the need for reading the full meaning into mathematical statements,—reading into them not merely the words and symbols, but the relations involved. Illustrations are given.

Gray, W. S.: and others: REMEDIAL CASES IN READING: THEIR DIAGNOSIS AND TREATMENT. Univ. of Chicago, Supp. Educ. Monographs, no. 23, Nov. 1923.

An extensive and technical monograph of 157 pages containing some usable recommendations.

Gray, William S.: READING AND PUPIL DEVELOPMENT. (Proceedings of the conference on reading held at the University of Chicago. Vol. II). Univ. of Chicago Press, 1940, pp. 36–41; 83–89; 138–142; 188–193; 253–265.

This is without question the most rich and helpful source of inspiration and suggestion encountered by this reviewer in relation to our present problem. The entire volume of 350 pages is well worth reading. The pages noted above seemed especially helpful in relation to the improvement of reading mathematical material.

Jablonower, J.: "The Jabberwocky Was a Lark—Or the Mathematician Takes a Holiday." *M. T.*, Vol. 26, May 1933, pp. 302–306.

An amusing and thought-provoking little dissertation, referring to the use of words in mathematical writing.

Jackson, Nelson: "Vocabulary in Beginning Algebra." *S.S.M.*, Vol. 35, Oct. 1935, pp. 690–694.

Description of a study showing students' deficiencies in the vocabulary of algebra, but there are no concrete suggestions for improving the situation.

Jacobson, P. B.: TWO EXPERIMENTS WITH WORK-TYPE READING EXERCISES IN NINTH GRADE. University of Iowa, Studies in Education, Vol. 8, no. 5, 1933.

Report of an experimental study of the effect of specific reading exercises in algebra and other ninth grade subjects.

Judd, Charles H.: EDUCATION AS CULTIVATION OF THE HIGHER MENTAL PROCESSES. New York; The Macmillan Company, 1936, pp. 91–93.

Description of an experiment illustrating types of reading difficulties encountered by students in translating verbal problems into algebraic symbols.

Judd, C. H.: THE PSYCHOLOGY OF SECONDARY EDUCATION. New York; Ginn & Co., 1927.

pp. 93–94: Discussion of the difficulties in translating verbal problems into algebraic symbols.

pp. 118–123: On the value of verbal discussions of mathematical definitions and ideas, symbols, axioms, and postulates.

Kempner, Aubrey: "Extension of Concepts in Mathematics." *M. T.*, Vol. 16, Jan. 1923, pp. 1–23.

A good helpful article well worth reading more than once.

Livingston, Ralph: "The Interrelations of the Vocabularies in Public-School Subjects." Educational Research Bulletin (Ohio State University), Vol. 5, May 12, 1926, pp. 208–214.

A statistical study of vocabularies in several school subjects. Attention is called to the fact that often different meanings are attached to the same word in different subjects.

McKim, Margaret Grace: THE READING OF VERBAL MATERIAL IN NINTH GRADE ALGEBRA. New York; Bureau of Publications, Teachers College, Columbia University, 1941, pp. 1–25; 76–84; 91–132.

This is a doctoral dissertation. Parts of it will prove too technical to be of much general interest but the sections included in the pages indicated above should be both interesting and helpful to classroom teachers of mathematics. There are many hints, both suggested and implied, for improving the ability of students to read verbal problem material.

Minnick, J. H.: TEACHING MATHEMATICS IN THE SECONDARY SCHOOL. New York; Prentice-Hall, 1939, pp. 116–117.

Suggestions for helping pupils overcome difficulties in reading problems.

Moore, W. and Pressey, L.: "Growth of Mathematical Vocabulary from the Third Grade through the High School." *School Review* Vol. 40, May 1932, pp. 449-454.

Stresses the importance of adequate technical vocabulary for meaningful reading of mathematics, but offers no detailed suggestions for achieving this adequacy.

Morley, R.: "You Know What I Mean." *M. T.*, Vol. 19, Nov. 1926, pp. 385–394.

On making more clear the meanings of the words and symbols used in mathematics. Pointed, entertaining, sensible, and helpful.

Myers, George W.: "Does Ft. $\times$ Ft. = Sq. Ft.?" *S.S.M.*, Vol. 27, Jan. 1927, pp. 69–74.

A good common-sense discussion of a question that sometimes is troublesome to students. Has a bearing on the interpretation of problems involving data based upon measurement.

National Committee on Mathematical Requirements: THE REORGANIZATION OF MATHEMATICS IN SECONDARY EDUCATION. (1923), Chapter VIII.

Terms and symbols of elementary mathematics are here given clear meanings and a degree of simplification and standardization. This chapter serves as a good standard of reference.

National Council of Teachers of Mathematics, Fifteenth Yearbook: THE PLACE OF MATHEMATICS IN SECONDARY EDUCATION. (1940), pp. 232–237.

Appendix III on "Terms, Symbols and Abbreviations in Elementary Mathematics" covers 15 pages. Stimulating and helpful in respect to vocabulary. Authoritative recommendations for present-day practice in the use of mathematical terms, symbols, and abbreviations.

National Council of Teachers of Mathematics, Eighteenth Yearbook: MULTI-SENSORY AIDS IN THE TEACHING OF MATHEMATICS. (1945).

Not directly on *reading* in mathematics, but very suggestive of ways for making mathematical terms more vivid and real, and giving motive for reading. Every mathematics teacher should have access to this book.

Orleans, J. B.: "Reading in the Teaching of Mathematics." *M. T.*, Vol. 32, Dec. 1939, pp. 349–351.

Contains brief but detailed illustrations of procedure in helping students learn to read algebra and geometry material better. A good article.

Osburn, W. J.: "Reading Difficulties in Arithmetic." Madison, Wis.; State Department of Public Instruction, 1925.

A pamphlet designed to give suggestions to teachers for analyzing pupils' reading difficulties in arithmetic and helping them to overcome these difficulties.

O'Toole, A. L.: "Remedial Reading in College Mathematics." *National Mathematics Magazine*, Vol. 14, March 1940, pp. 329–344.

A comprehensive and valuable article in which the author analyzes some difficulties involved in reading mathematical matter, and reviews and summarizes various studies. An extensive bibliography is presented at the end of the article.

Overman, J.: "Teaching the Algebraic Language to Junior High School Students." *M. T.*, Vol. 16, Apr. 1923, pp. 215–227.

A somewhat long article but one which contains helpful suggestions.

Pressey, L.: "Words of Unusual Derivation in Common Mathematics Vocabulary." *M. T.*, Vol. 19, Oct. 1926, pp. 341–348.

A list of 129 mathematical terms with their derivations.

Pressey, L. C.: MANUAL OF READING EXERCISES FOR FRESHMEN. Columbus, Ohio; Ohio State University Press, 1928.

A book of exercises for improving the reading ability of college freshmen. Suggestions are given with the exercises.

Pressey, L. C. and Elam, M. K.: "The Fundamental Vocabulary of Ele mentary School Arithmetic." *Elem. School Journal*, Vol. 33, Sept. 1932, pp. 46–50.

Description of a procedure that could be used by teachers to improve reading comprehension in arithmetic. Brief, readable, and worthwhile.

Pressey, L. (and others): "Essential Vocabulary in Algebra." *S.S.M.*, Vol. 32, Sept. 1932, pp. 672–674.

A list of "Essential Terms" is presented with a suggestion of how to test for recognition of them.

Progressive Education Association: MATHEMATICS IN GENERAL EDUCATION. New York: D. Appleton Co., 1940, pp. 77–240.

This whole section of the book is related more or less directly to the problem of reading mathematical material. While it does not attack the problem of reading directly, much of the discussion has an implicit bearing upon this problem.

Russel, D.: "Introducing Mathematical Concepts in the Junior High School." *S.S.M.*, Vol. 38, Jan. 1938, pp. 6–19.

An interesting article with concrete suggestions for developing and clarifying the meanings of mathematical terms and relationships.

Rutan, Edward J.: "Teaching the Language of Mathematics." *The Nation's Schools*, July 1945, pp. 43–44.

This article, by a teacher of English, suggests ways in which English

teachers and mathematics teachers might work together to improve reading comprehension in mathematics. The suggestions and possibilities are worth careful consideration both by teachers of mathematics and teachers of English.

Sampson, C.: "Building a Foundation in Mathematics." *Journal of Educational Method*, Vol. 2, Oct. 1922, pp. 76–78.

Brief but good suggestions for using materials such as flash cards to build geometric concepts.

Schaaf, W. L.: "Testing the Clarity of Mathematical Concepts." *S.S.M.*, Vol. 39, Oct. 1939, pp. 651–656.

Suggestions on detecting inadequate mastery of terms used in elementary mathematics. Suggestions could be useful in diagnosing reading difficulties.

Schorling, Raleigh: THE TEACHING OF MATHEMATICS. Ann Arbor, Michigan; The Ann Arbor Press, 1936, pp. 118–119; 220–222.

Suggestions for improving the ability of students to read algebra understandingly.

Stright, Isaac L.: "Relation of Reading Comprehension and Efficient Methods of Study to Skill in Solving Algebraic Problems." *M. T.*, Vol. 31, Dec. 1938, pp. 368–372.

Report of a study of the effect of a program of instruction in remedial reading on the improvement of students' ability to solve algebraic problems.

Welte, Herbert D.: A PSYCHOLOGICAL ANALYSIS OF PLANE GEOMETRY. University of Iowa Monographs in Education, Iowa City, Iowa, Jan. 1926, pp. 17–27.

This study presents a detailed and exhaustive list of 600 words which, in the author's opinion, represent the technical vocabulary of plane geometry. It also contains a list of 288 "main ideas of plane geometry." These include definitions, axioms, postulates, and some basic propositions and could be very helpful in clearing up hazy ideas.

HOW CAN THE READING OF CONTEXT AND EXPLANATORY MATERIAL BE IMPROVED SO THAT THE STUDENT WILL HAVE A CLEAR UNDERSTANDING BEFORE PROCEEDING INTO ACTUAL PROBLEM SOLVING?*

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THE PROBLEM

One of the most striking characteristics of modern mathematics textbooks is that they contain prefatory discourses. Textbook writers feel that this is important at the beginning of almost every new unit, chapter, or even sections. The purpose of

* Presented to the Junior High School Group of the Central Association of Science and Mathematics Teachers, November 24, 1945.

such discourses is generally four-fold: (1) to give the student an orientation to the new work ahead; (2) to provide him with adequate motivation in his exploration of new paths; (3) to present suitable explanation of new terminology; and (4) to demonstrate by analysis or synthesis new processes. This introductory feature usually serves as the basis of any impression made on the student, good or bad. As a means of helping the student derive the optimum benefits from these contexts, teachers of mathematics should teach their students the skills and techniques of reading in this specific area. The purposes of this discussion are: (1) to describe the kinds of materials used to introduce new concepts; (2) to demonstrate the need for the teaching of reading of these contexts, (3) and to offer suggestions to textbook writers and teachers of mathematics for the improvement of such materials.

In making textbook analyses of the kinds of materials used to introduce units or chapters, the writer found after having examined some twenty-five arithmetic textbooks taken from the curriculum laboratory of the University of Chicago, the following types of explanatory material: (1) context, that is the relationship as expressed by ideas, an aspect of this problem adequately discussed by a previous discussant; (2) explanatory material in simple discourse; (3) the inductive development of concepts and processes and lastly, but by no means insignificant, (4) principles which call for a realization of the nature of generalizations, in addition to the giving of reasons for the truth or validity of the principle involved. Moreover, an analysis of the kinds of reading behavior involved in the effective comprehension of these kinds of material revealed that among many, the six kinds of reading were encountered: (1) reading to obtain facts; (2) reading to analyze facts; (3) reading to interpret facts; (4) reading to understand the mathematical processes; (5) reading to understand the subject matter vocabulary; and (6) reading to see the implications for application in similar problem situations.

What Does a Typical Classroom Situation Show?

In an effort to combine the fruits of experience with theory, a study was made by this discussant last week to define the difficulties normally encountered in the reading of the materials used to introduce units in arithmetic on the Junior High School level. A group of thirty-six (36) seventh grade students was

presented a unit in a popular textbook on the division of decimals. The dividend and the divisor contained decimal fractions. Questions were framed in such a fashion as to check on the pupils' abilities to do the six kinds of reading referred to in this paper. The following are the introductory remarks, the sample problem, and the first problem of this particular type to be worked by the students presumably without any teacher-help:

"The children are selling tickets for the Harvest Festival. The money is to be used to buy playground equipment. The sixth grade received in all $16.90 for the tickets sold. If the tickets sold for $0.30, how many tickets did the class sell?"

The following table lists the principal kinds of reading essential for the solution of the problems of this type, the percent of the group who answered questions correctly and incorrectly.

READING OF EXPLANATORY MATERIAL IN ARITHMETIC

Kinds of Reading	Percent Correct	Percent Incorrect
Obtaining Facts	42	58
Analyzing Facts	47	53
Interpreting Facts	79	21
Understanding processes	79	21
Vocabulary	22	78
Seeing Implications	33	67

These data show that the students encountered difficulty in the reading of these two arithmetical problems. It is obvious that they were not acquainted with the reading skills prerequisite to the actual problem solving; the vocabulary was not understood, they were unable to obtain the facts and in more than one-half of the instances chose irrelevant and non-essential factors in their analysis of the facts. In their efforts to interpret the facts, they did not have clear ideas with respect to the distinction between the facts given and the facts to be found. It might be argued that division of decimals by decimals should be introduced with much teacher-aid. However, the division pattern found in dealing with whole numbers was present and should have had sufficient transfer elements. This indicates that either the teacher did not emphasize the pattern sufficiently or that the children did not recognize the similarities.

The most serious disability possessed by this group was that it did not see the implications for application in similar types of exercises. The purpose of the sample is to make it easier for these pupils to recall the simple action patterns for which the

experience with the sample should provide. This should serve as a basis for their reorganization of the elements selected so that they might make appropriate application in similar situations.

Suggestions for Textbook Writers

Textbook writers can do much to improve context and explanatory material in the first place by their selection of problems to serve this purpose of introduction to new units. The usual criteria of desirable problems should be kept in mind: (1) genuineness, (2) cruciability, (3) modern setting, and (4) an appeal to the interest of the child. The explanatory problem should set the ball rolling.

A second suggestion to textbook writers is with regard to number of symbols per sentence, both verbal and numerical. The writers of textbooks should take cognizance of the fact that eye movements are much more frequent in the reading of numbers than in the reading of words. When problems are long, complicated, and contain several numbers, even experts have to re-read them in order to place the numbers in their proper relation.

A third suggestion to textbook writers is that they should exercise great care in their selection of vocabulary. The vocabulary of the textbooks should not differ too widely from the vocabulary of the children who read them. Even though the teacher must assume responsibility for teaching the meanings of new words as they occur, their tasks can be more simplified by the exercise of care on the part of textbook writers. More suggestions could be given to textbook writers but others on this panel have pointed out areas applicable to these writers. The three suggestions made seem pertinent.

Suggestions to Teachers

Experienced teachers know that much of the high mortality in arithmetic is due to defective reading on the part of the students. Further, they know that the textbooks are written for their use and that if the books are not satisfactory, they must make them so. They have been reminded time and again that they are greater resources than any textbook selected by or for them! This improvement of the material must be the concern of the mathematics teacher.

These suggestions were within the scope of the discussion—

developing the ability to recognize relationships in the reading of problems, the use of suggested aids and references by teachers, and the development of ability to read symbolic language. In addition the teacher must develop vocabulary meaning by extending the sense meaning of words, by fusing meanings into related ideas, by developing the ability of students to react critically to the words and having them see implications. Among many techniques at her command she might ask questions which will help the students to interpret the printed page in the light of their experiences with the word and extending these experiences with the context at hand.

Another suggestion to teachers is that they teach children to critically analyze the problem so that the vital question, "How does it apply?" might be answered. In addition to the teaching of skills in the performance of the arithmetical operations in these explanatory problems the teacher should stress applications to other situations. They must spend more time at this point of the game or cause discomfort to their students.

In summarizing, the problem has been defined, the types of explanatory and context materials have been given, a survey of the status of a typical classroom situation with sample problems to check on the students' abilities to do the types of required reading has been presented, suggestions have been given to textbook writers and to teachers to overcome the difficulties involved in context and explanatory materials. They are specific responsibilities which must be met. Are we willing to assume these responsibilities? That is the question.

IN WHAT WAYS CAN THE MATHEMATICS TEACHER DEVELOP A MEANINGFUL MATHEMATICS VOCABULARY AND NOT DESTROY THE MEANING THOSE SAME WORDS MAY HAVE IN OUTSIDE SITUATIONS.*

BERNICE BLAKELY
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"Vocabulary in arithmetic?" (With raised eyebrows in a skeptical tone.) "There is so much to teach, I don't have time for word study" or "Let the English teacher do the word meanings, I have all I can do with the mathematics," were some of

* Presented to the Junior High School Group of the Central Association of Science and Mathematics Teachers, November 24, 1945.

the answers I got when asking the question, "How do you teach word meanings in mathematics?" But I also got the reply "When a word has been in use for a long time and grown rich with associations in many situations, it carries meaning and the mathematics teacher will not have to worry about the meaning being destroyed in outside situations."

I don't think there is a person who would dispute the fact that a child will have to understand the words if he is to read a problem intelligently. Teachers seem to be in fair agreement that a reading readiness program is essential. The child needs experiential background to help build up a vocabulary so he will be able to express his ideas and eventually get meaning from a printed page.

In the field of numbers we have been slower to accept this idea. Too many still cling to a goal that means mastery of certain computational skills with the feeling that if a child has mastered them he can use them in any situation. The ability to do the quantitative thinking doesn't follow and we have children floundering and teachers anxious or disgusted with their own efforts and everyone else's.

In real life we do not figure on problems about which we know little or nothing. We consult an expert or read up on the subject whether it be home building, electric wiring, making slip covers for furniture, knitting, or building feeding troughs for pigs. When the sixth grade was ordering lumber for a feeding trough for the purebred hereford pig, they consulted a pig magazine on the subject for dimensions. One little boy multiplied length, width, and depth together because he said that was what he did when he had three numbers. He was surprised to find there would be no room for the poor pig's food! A problem is put in quite a different light when it is to be carried through and not be just an answer on paper that says it will take 5 days to do the work, or cost $2.50, or we will need 5 board feet of lumber. Too often we have a page of problems each one of which pertains to a different situation so that a child may have to deal with ten different subjects for which his information is very hazy. He feels he must have an answer and so does something with the numbers. We find, however, that textbooks have improved a great deal in that there is more of an attempt to give some background and then have a set of problems dealing with a particular situation.

Perhaps informational units would be one way of getting

ideas across in situations as near real as possible. They help to put meat on the bones of numbers. One or two periods spent in discussion and experiencing will furnish about all the problem material a class could handle. New words and ideas are taken up in their natural setting.

Miss Hennigar in her thesis on "Informational Units for Sixth Grade, the Family Budget" found that new vocabulary, although no technical drill was given as such, made quite an impression on pupils as was evidenced by their use of new words whenever an opportunity presented itself. Some words were budget, necessity, advancement, miscellaneous, eliminate.

Suppose you are working on banking. You learn that when money is put into the bank it is "deposited." This sounds very simple. However, when you go to the bank you must choose the correct kind of deposit slip according to your particular kind of account. It must be filled out properly, taken to the right window, and a bank or deposit book presented otherwise a receipt for the deposit should be given. If you are opening an account, another procedure must be followed. Knowing what the word "deposit" means isn't enough but ideas about its use are necessary. The quickest, surest way to help a child with these understandings is to have him actually experience it by a trip to a bank and having a bank in the room so he can practice banking. One needs no elaborate equipment, no extra periods, no excuse for large classes.

I read about one group that organized a company and insured fountain pens. Did they know what a policy was, a premium or commission? They surely did because they made the policies themselves, decided on the amount of premium and used their powers of salesmanship on boys and girls in grades five to eight.

One teacher told me about using Sears Roebuck catalogues. The class discussed making selections, wise choices, using good judgment, advantages and disadvantages of ordering by mail. They made out orders. Quantity, quality, amount, zones, freight charges, express, weights, money orders, personal checks, and many other terms they knew because they had to use them.

When working on measure of any kind, actually measuring things will insure most learning. Rather than learning a table of measure, let children experiment and find out how many cubic inches a cubic foot contains by fitting inch cubes into a box the size of a cubic foot; draw a square yard on the floor to see and walk over; cut a tin can apart and get the area of the circle and

see that the sides are really a rectangle when flattened out. Dimensions, length, width, area, volume, radius will be more real because they see them, feel them, walk around them, and measure around them.

If the children did more activities themselves, we wouldn't have such confusion as Stevenson reported in his investigation of "Difficulties in Solving Problems" when children in grades five to eight were asked what area meant—"The sum you get when you add different land; number of people who live in our country; boundary of the United States." Then there is the oft quoted statement that "average is the thing hens lay their eggs on."

There is a question as to how much value there is in specific vocabulary training in arithmetic. Harry Johnson's study "Effect of Instruction in Mathematics Vocabulary Upon Problem Solving in Arithmetic" found that

1. Specific words studied showed improvement in understanding but little evidence of transfer to other words.

2. There was improvement in problem solving where those particular words were used.

He used—Daily oral drills—Dictionaries—Individual notebooks—Mimeographed material.

Following are some samples: 1. Words referring to common topics were grouped and relationship discussed. *Area* refers to *surface* and is measured in *square units* such as *square inches*.

2. Opposite each write volume, area, or perimeter

350 sq. ft.	3 sq. in.
40 sq. yds.	35 cu. in.

3. Tell whether each of the following refers to area, perimeter, or volume,

size of county or state
truck load of sand

Brueckner, in his "Diagnostic and Remedial Teaching in Arithmetic" suggests:

1. Ask pupils to restate a sentence using other than those words underlined. Mr. Reed sold a car for Mr. Wells at a *commission* of 10%. Mr. Reed *endorsed* the check.

2. Have problems without numbers so the child can concentrate on ideas.

Other Methods

1. An automobile salesman sold a car for $1200.00. He re-

ceived \$60.00 or 5% for his sale. What is \$60.00 called?

2. Have the children restate the problems in their own words.

3. Give some word terms or phrases and have the pupils construct their own problems as—simple interest, annual, premium.

In summing this up, I find nothing new nor startling has been offered but briefly I should like to say:

1. First of all nothing can take the place of a good teacher, one who is interested in teaching *children* the mathematics rather than teaching *mathematics* to the children; one who recognizes individual differences and does something about them; one who knows a great deal about quantitative thinking as well as computational ability.

2. Teach a new idea, word, or term slowly, giving the children a chance for a great many experiences and associations so they really have meaning. It isn't how fluently a child can give back definitions that tells you how well he has grasped an idea but how wisely he can use and how intelligently he can discuss the information.

C. E. Ragsdale has said in *Directing Learning*. "There is a clear recognition that an educated person will always be able to think best with respect to those categories of knowledge concerning which he has the widest range of information and especially the most practice in thinking."

3. Perhaps less departmentalization would help overcome the feeling on the part of both teacher and pupil that certain things can be put into compartments and kept there. Our lives aren't made that way. We use reading, English, mathematics, science all at the same time in solving almost every problem. Every teacher has to teach a little of everything with special emphasis on the technical terms peculiar to the particular subject predominant in the problem.

4. In conclusion I should like to repeat "When a word has been in use for a long time and grown rich with associations in many situations, it carries meaning and the mathematics teacher will not have to worry about the meaning being destroyed in outside situations."

Man is an ordinary creature—in height about six feet tall—near the geometric mean between the distances the astronomers talk about, say thousands of light years, and the dimensions of an atom, where a glass of water contains about 20×10^{24} atoms.—Robley D. Evans.

AN EXPERIMENT IN ADOLESCENT INGENUITY

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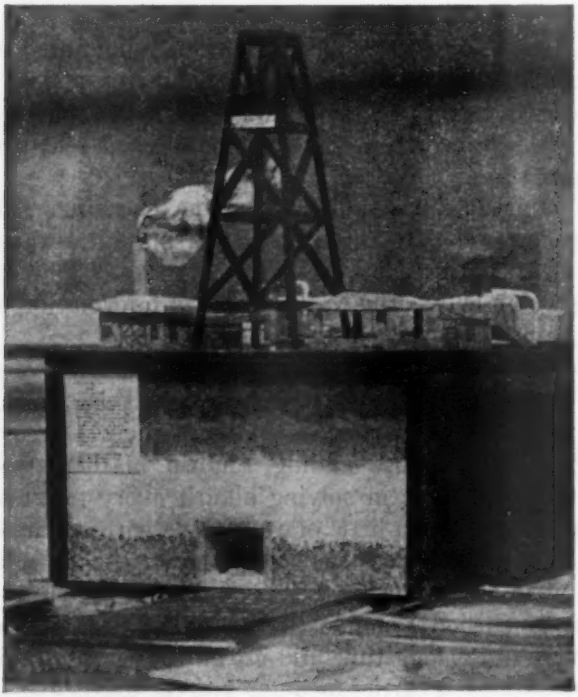
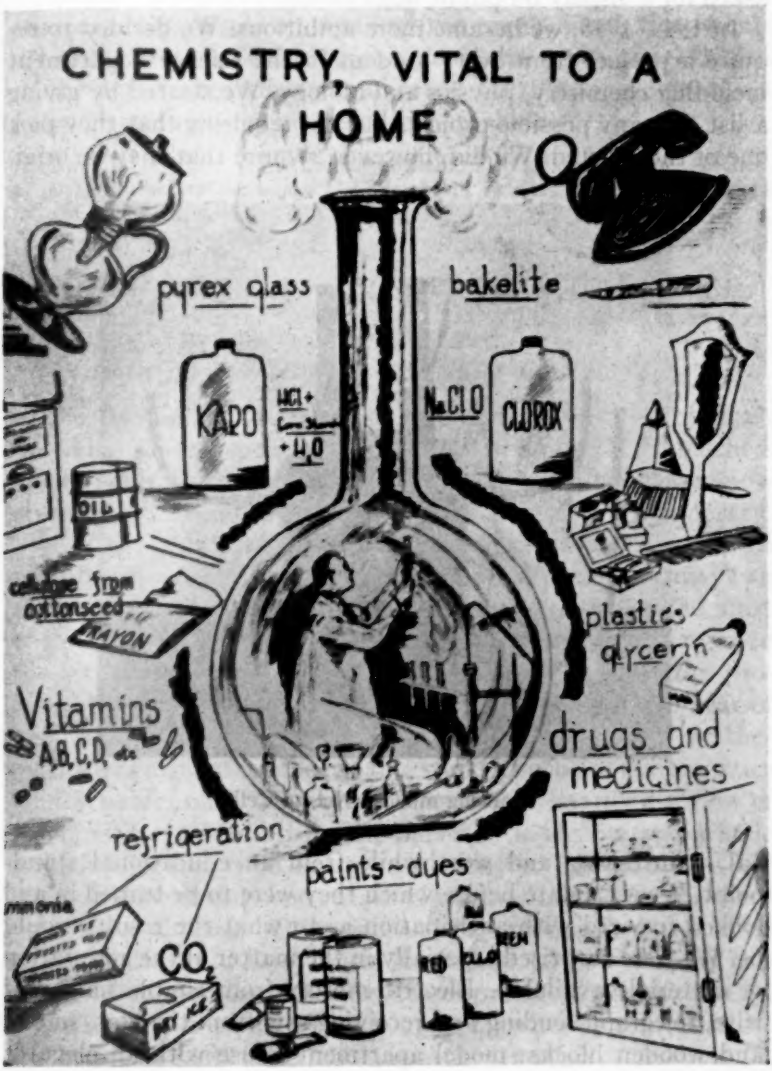
About twenty years ago, after our Chemistry Club was solidly established, some of our members started making projects. The motto of the club was "never a half year without something constructive accomplished." Our first outstanding project was a working model of the contact sulphuric acid plant. This model is still in operation. With this start, we began other projects mainly of a kind that would become a permanent part of our science set-up. As time went on, the production of these projects



A model of the Frasch Process for getting sulphur.

led to our introduction of "open house" evenings with the public invited to see the projects in operation. The reaction of our visitors was always very favorable. However, with the start of the World War II, these "open house" meetings had to be abandoned.

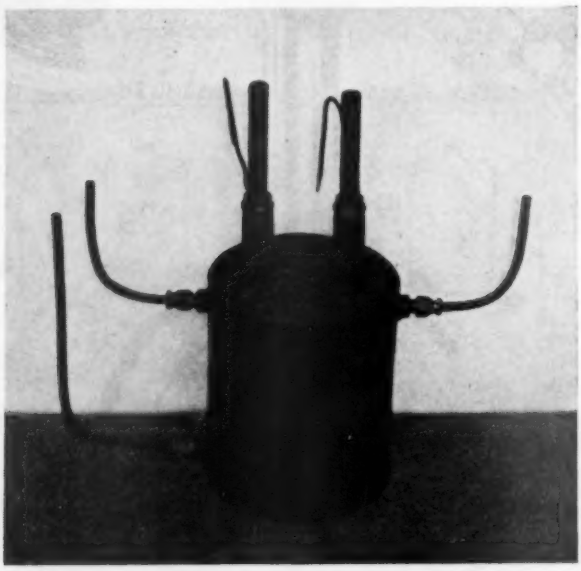
In 1933–1944, we decided to require a project from every student in our chemistry classes. It was successful. Between 250 and 300 projects were turned in, including working models of



the Frasch process and electric furnace, models of the steel and blast furnaces; booklets on metals, glass, plastics, vitamins, anesthetics and alloys; charts on cosmetics, inks, rubber (synthetic) and many other topics; and models of atoms and mole-

cules. Many of these were so outstanding we were asked to set up an exhibit in our cafeteria, so that all the pupils in the school could see what we were doing in chemistry. One of the banks asked for a display for public observation.

In 1944–1945, we became more ambitious. We decided to require a project from every student in the science department including chemistry, physics and biology. We started by giving a list of many possible projects but not requiring that they pick one of those listed. We did, however, require that they be origi-



Working model—Nelson Cell

nal, painstaking and worthwhile from an educational standpoint. We set a date before which they were to be turned in and looked forward with anticipation as to what the results would be. We were surprised especially in the matter of the adaptation of materials available. Electric motors from simple nails and wire, telegraph sending and receiving sets from tin cans, screws and wooden blocks; model apartment house with an elevator (working model), a jet propulsion plane (it works), simple radio sets, model of the human head (vertical section), chemical balance, a working bleachery, electrolytic cell, open hearth furnace, ionization apparatus, and a working model of the blast furnace were just a few outstanding examples of the youngster's

work. As a result of our experiment we had exhibits not only in the school but in two of the banks and have been asked to put one in the local library.

We plan to continue requiring a project from all of our science students. We are sold on the idea. The pupils like it, the parents approve, we get valuable teaching aids, the teachers are enthusiastic and, as one of our youngsters said "when you make a project you surely know the process by heart."

(Editor's note—Dr. Walrath has consented to describe some of these projects in greater detail for the benefit of readers of SCHOOL SCIENCE AND MATHEMATICS.—W.A.T.)

DO WE NEED REFRESHER MATHEMATICS?

ALBERT R. MAHIN
Hartford City High School, Hartford City, Indiana

The experiences of the last five years have re-emphasized the value of mathematics in the school program. Most teachers of this subject are wondering whether the prominent place now given to mathematics is permanent or if it is only temporary. The author of this report can remember the time when all high school graduates in his school were required to take at least two years of mathematics—algebra and geometry. Until the start of the war the trend was in the opposite direction, and prior to the introduction of the war-time emergency mathematics program, the school in which he teaches required only one year of high school mathematics for graduation. This year of mathematics was algebra for the capable students general mathematics for the poorer ones. The latter course was primarily a review of the fundamentals of arithmetic plus a little algebra and a little intuitive geometry. In either case, many of our graduates left high school with only one year of mathematics, and for the three years prior to their graduation had had no mathematics. This report deals with a study made in our high school—a study primarily of our Seniors and their lack of proficiency in arithmetic. Being a study of a particular group, it may not be typical of the results which could be obtained in other schools.

Our school followed the general directives of the State Emergency Mathematics Committee as set forth in the bulletin, "Emergency Mathematics Program in Indiana High Schools." Shortly before the end of the first semester of the year 1942–1943 we gave the Standard Arithmetic Achievement Test (Ad-

vanced), Form D, to the Seniors and all other students who were seventeen years old or older. One hundred forty-two students took this test and ninety-four, approximately two-thirds of them, failed to make an equated score of 78, (an equated score of 78 was to be regarded as a passing score). We were astonished —and dismayed—at these results. It indicated a situation which needed a definite remedy.

The ninety-four students who had not passed were placed in so-called Refresher Mathematics classes. Two groups of them, seventy-four in all were taught by the author of this article, the remaining students were placed in commercial arithmetic classes taught by another teacher. For the latter students, commercial arithmetic was taught to the regular class for three days of the week, and the regular class, along with the refresher mathematics students had the special refresher mathematics course on the other two days. We decided to use the workbook "Arithmetic for the Emergency" by Ruch, Knight and Studebaker, taking one self-administering test and assigning one of the problem sets each week giving enough additional tests and problem sets towards the close of the semester so that all of the workbook would be studied. Those familiar with this workbook realize that adherence to its plan provides a large amount of remedial drill work for those who need it. In teaching, a definite attempt was made not only to explain the more difficult problems after each test and problem set but also to have the student be able to identify the process in such a way that it could be used in solving other problems of a similar nature.

A rather complete study of the results of the Stanford Test, Form D, was made, and additional tests were given during the semester. These include "A Screening Test in Arithmetic," by Brueckner, at the close of the first six weeks, "The Cooperative Achievement Test in Arithmetic, Form Q," and the "Stanford Test, Form F," at the close of twelve weeks, and as a final test at the end of the semester, the Stanford Test, Form E. Most of these students had also taken an intelligence test. Therefore the results of the tests were tabulated along with the Otis I.Q. of each student. These I.Q.'s for 92 students ranged from a high score of 116 to a low score of 67. Such scores were not available for the remaining students most of whom were members of the commercial arithmetic class. Although one would naturally expect those with lowest I.Q. scores to make the least improvement during the course this was not universally true. One

student with an I.Q. of 108 failed to make sufficient improvement to pass while the student with the lowest I.Q., 67, improved enough to pass the course. Isolated examples, however, do not show complete results. Seven persons with I.Q.'s of 100 or more did not pass, seventeen persons with I.Q.'s between 90 and 99 did not pass, and only four persons with I.Q.'s below 90 made passing scores.

Although Table I gives the final results. Table II giving the number right, number wrong, and number not worked for each problem of the Stanford Test, Form D, was more interesting to the author of this report. These tables are not included but a summary reveals some of the interesting facts. Of the one hundred forty-two students who took this test, the forty-eight who passed averaged 24.75 problems right, 6.85 problems wrong and 8.40 problems not worked on the reasoning portion of the test; those who did not pass averaged 12.10 problems right, 10.63 problems wrong and 17.17 not worked. The average for all was 16.45 right, 9.35 wrong and 14.20 not worked. In order to get a passing score of 78 on this part of the test it was necessary to work 19 problems correctly, yet six students worked fewer than 19 problems and one of these six attempted to solve only 10 problems. On the computation section the averages were as follows: for those who passed, 47.35 right, 10.90 wrong, and 6.75 not worked; for those who failed 22.94 right, 17.26 wrong and 24.80 not worked; and the average for the entire group was 31.19 right, 15.11 wrong and 18.70 not worked. On this section of the test where 39 correct problems were required for an equated score of 78, there were 13 students who worked less than 39 problems. While on the reasoning test, the entire group averaged 63.8% correct problems of those they worked, the 48 who passed had 78.3% right and the other 94 averaged only 53.2% correct. On the computation test the results were similar. The entire group averaged 67.3% correct problems, the 48 who passed had 81.3% right, and those who failed had 57.1% right. These results indicated that those who failed did not work enough problems in the allotted time, and it also indicated that they made errors on too many of the problems which they worked.

A further analysis of the 105 problems of the test was made to determine whether they were worked correctly, not worked, or if worked incorrectly the type of error made. It was discovered that on the reasoning test the greatest number of errors was

due to the use of the wrong process, (688 errors), and next in order was errors in computation, (349 errors). Besides these two causes which accounted for 1037 of the 1328 errors the following were listed as causes of errors; misinterpretation of the problem, writing the answer in the wrong form, (a decimal for a fraction, etc.), not completing the problem, (working part of it correctly and giving an answer without taking the final steps in the solution), and having the right numerical answer with the decimal point in the wrong place. On the computation test also the greatest number of errors was due to the use of the wrong process (1097 errors). Next in order was errors in computation, (548 errors), and the third in order was putting the decimal point in the wrong place, (315 errors). These accounted for 1960 of the 2146 errors. Other causes of error were having the answer in the wrong form, having the problem incomplete, and giving the answer the wrong algebraic sign. This information was in accord with our previous conclusion that one of the greatest causes of loss of time in a test is trying to determine the process to use in the solution of each problem. Since so many errors were due to use of the wrong process it seemed likely that a considerable part of the time had been used in trying to select a process for solving the problems.

To answer the question what kind of problems were missed, the table was checked to find out what problems had been missed by thirty-five or more students. It was found that on the reasoning test there were eighteen such problems, and on the computation test twenty-seven problems. To analyze the errors in these 45 problems, they were examined to determine the processes involved in their solution, and also the most common error for each problem. These are listed in Table V. A close study of these problems reveals that thirteen of them involved per cent or percentages and the chief cause of error in ten of these problems was the use of a wrong process. The students apparently did not know how to solve percentage problems. Eleven problems involved division as the principal process. In these problems errors in computation were chiefly responsible for mistakes although in the division of fractions there was a failure to invert the divisor, and another very common source of error was putting the decimal point in the wrong place. These two kinds of problems accounted for more than half of the problems in this table. Other arithmetic processes either used at the wrong time or in the wrong way accounted for most of the other errors.

In area problems which required changing from square feet to square yards the most common error was dividing by the number of feet in a yard rather than the number of square feet in a square yard.

From the study of this test the author concluded that these students made many of their mistakes because they did not understand the processes involved and could not apply them correctly. Although they did not work enough problems this was probably (in many cases at least) due to a lack of understanding. This resulted in a considerable waste of time in trying to determine a method for solving the problem.

At the close of the first six weeks we gave "A Screening Test in Arithmetic" by Brueckner to our Refresher Mathematics classes and also to all other high school mathematics classes in our school, a total of 288 persons taking the test. This test consist of thirty problems including multiplication, addition, subtraction, and division of whole numbers, decimals, and fractions, and the three general types of percentage problems. The average number of problems worked correctly was 16.61 out of the thirty problems. One student made a perfect score, while two students, both freshmen, had only three problems right. The entire freshman class took the test and their average was 13.80 right as compared to the Refresher Mathematics students who averaged 17.71 right. One group of juniors and seniors in a solid geometry class had an average of 27.00.

The results of this test were also analyzed to determine the number and types of errors on each problem. In tabulating results, the problems were listed as correct, not worked, or wrong because of error in computation, wrong process or wrong position of the decimal point. The totals revealed that, arranged in order of the number of errors, the types of problems should be; percentage, division, multiplication, subtraction and addition, the last one being the least difficult process. Percentage problem errors were due to the use of the wrong process with the various cases being mis-applied very frequently. In the percentage problems, too, there were more errors by misplacing the decimal point than in any of the other types of problems. Division and multiplication problems were next as a source of error. Twenty more problems in division were either not worked or worked incorrectly than in multiplication although the greater proportion of the errors in multiplication problems was due to errors in computation. Twenty-eight more errors in division problems

TABLE V.—TYPES OF PROBLEMS MISSED MOST-COMMON ERRORS

Problem No.	Processes Involved In Correct Solution	Most Common Error
	PART A—ARITHMETIC REASONING	
11	Division by 4. Addition	Dividing by 3 instead of 4
23	Cost+Expenses—Selling Price, % of Loss on S.P	Dividing Loss by Cost instead of S.P.
28	Find Interest for 3 mo. Add to Principal	Treating 3 mo. as ¼ yr. Not Adding
4	Multiply Stops per Hr. by No. of Hrs.	Not Finding Stops per Hr.
13	Percentage—Case II	Answer Given as Fraction
17	Finding Total Time from P.M. to A.M.	Incorrect Addition
20	Division. Answer as Decimal Fraction	Finding Increase Using Wrong Divisor
9	Add. Divide to Find Average	Incorrect Addition
27	Find Area. Dimensions in Feet. Ans. in Sq. Yd.	Dividing Sq. Ft. By Feet in 1 Yd.
7	Division. Mixed Number by Fraction	Mixed No. Not Changed to Right Fraction
12	Division. Whole Number by Fraction	Multiplying Instead of Dividing
14	Multiplication. Dollars by Decimal	Subtracting Decimal—Not Multiplying
18	Find Two Areas. Subtract One from Other	Subtracting Dimensions
6	Percentage—Case I	Error in Multiplying by Mixed Number
15	Area of Trapezoid—Apply Formula	Finding Product of Bases
22	Percentage—Case III	Multiplying by % as Whole Number
5	Find Volume of Rectangular Solid	Errors in Multiplication
29	Find Ratio of Decimal Numbers	Failure to Give Simplest Form
	PART B—ARITHMETIC COMPUTATION	
27	Division of Whole Numbers	Not Placing Zero's in Answer
38	Percentage—Case I	Not Inserting Decimal Point
32	Percentage—Interest for 3 Mo.	Treating 3 Mo. as ¼ Yr. or as 3 Yr.
33	Percentage—Case I (Over 100%)	Decimal in Wrong Place
24	Fraction Times Mixed Number	Errors in Cancellation
19	Division—Fraction by Whole Number	Failure to Invert Divisor
22	Division—Whole Numbers	Not Placing Zero's in Answer
37	Percentage—Case II. Finding Interest Rate	Dividing by Wrong Number
12	Changing Decimal to Per Cent	Dividing Decimal into 100
26	Division. Decimal by Decimal	Decimal in Wrong Place
21	Addition. Mixed Numbers	Error in Adding Fractions
8	Multiplication. Fractions	Adding Denominators
30	Division. Decimal by Decimal	Error in Dividing
45	Percentage. Chain Discount. Find List Price	Taking Both Per Cents Times List Price
31	Subtraction. Denominate Numbers (Yd. Ft.)	Failure to Borrow First
47	Algebraic Subtraction	Adding Numbers
42	Find Area. Dimensions in Feet. Ans. in Sq. Yd.	Dividing Sq. Ft. by Feet in 1 Yd.
20	Addition. Mixed Decimals	Errors in Adding
11	Multiplication. Decimals	Placing Decimal in Wrong Place
41	Volume of Rectangular Solid	Adding Dimensions
16	Change Per Cent to Fraction	Dividing Per Cent into 100
17	Change Fraction to Per Cent	Writing Numerator as the Per Cent
4	Subtraction. Mixed Number from Whole Number	Placing Fraction in Answer without Subtracting
28	Division. Whole Numbers	Errors in Division
13	Subtraction. Mixed Decimals	Placing Decimal at Wrong Place
23	Addition. Denominate Numbers (Min.-Sec.)	Failure to Reduce Answer
25	Division. Mixed Numbers	Errors in Changing Numbers to Fractions

were due to use of wrong procedures (principally failure to invert a fractional divisor), than multiplication problems. There were also 81 decimal errors in division as compared to only 25 such errors in multiplication.

In the multiplication problems more errors were made on fractions than on either whole numbers or decimals. Although it seems that this indicates more difficulty with fractions, this is not necessarily true because the decimal fractions being last were not worked as often, and had they been worked might have been responsible for the greater number of errors.

From these tests, one of which was given to only a few besides seniors, the other of which was given to some members of each high school class, certain conclusions seem to be justified. Percentage problems and division problems are more likely to be missed than any of the other types of problems. Percentage problems of any type are difficult, and division problems, especially those involving either fractions or decimals are next in order of difficulty. Problems which require the use of fractions in any of the four fundamental processes are more difficult than similar problems requiring only whole numbers. The principal difficulty in decimal problems is placement of the decimal point.

At the close of twelve weeks, two more tests were given; "The Cooperative Achievement Test in Arithmetic, Form Q," and the Stanford Test, Form F. The first test was given primarily to provide additional training in taking a standard test. The 119 students who took this test had an average of 51.93, which was slightly higher than the end-of-year mean for ninth year students. Form F of the Stanford Test was given to determine the amount of improvement for each of the students, or how much they would need to improve to obtain the necessary score of 78. The average score on this test was 74.41. As already mentioned regular members of the commercial arithmetic class also took this test. Members of the Refresher Mathematics classes who were taking their second Stanford test made an average improvement of 9.52, and only three of them made a lower score than on their first test.

Finally, at the close of the semester, the last Stanford Test, Form E, was given. In its administration we departed somewhat from the standard procedure. Each student was given a colored pencil and after working the regularly allotted time for each part with his regular pencil he was given an extra ten minutes to work using the colored pencil. In this way a comparison of the

amount accomplished in the regular time and in the overtime period could be easily made. The scores given in Table I are the scores made in the regular time. The average score for this test was 78.40, an average improvement of 11.60 per pupil in the period of eighteen weeks. Although in some cases pupils made a lower score than on Form F, in no case did a student who had passed on Form F fail to pass on Form E. Of the ninety-four students having failed the original test, fifty made scores of at least 78 on the final test. Sixteen others unable to work enough problems in the regular time raised their score to 78 or higher in the overtime period.

Considering those who made scores of 78 or higher in either the regular time or in the overtime period to have passed, the record is still only fair. Of the 142 students taking the original test, only 48 passed. Of the 94 taking the Refresher Mathematics course, 66 passed. Percentage results indicate that only 80.3% of our Seniors could pass the original test, or pass after a semester of work. Stated another way, 19.7% of our Seniors were so poor in arithmetic that they could not make a satisfactory score on a test even after a semester's work in arithmetic.

We had to make certain changes in our mathematics program. All freshmen now take algebra, not necessarily because this is the best procedure, but because it is difficult to secure a general mathematics textbook which is adequate, and a teacher who is really sold on the teaching of general mathematics. Also the guidance problem is less difficult. We also have instituted a four year mathematics course for those who desire it, including two full years of algebra, one and one-half years of geometry, and one-half year of trigonometry.

We still give the Stanford Achievement Test, Form D. The state directive has been revised so that a score of 72 is now regarded as satisfactory so we require all students who do not obtain this score to take the Refresher Mathematics course two days each week. The test is given at the end of the sophomore year for each student in our school. Those who do not pass may take Refresher Mathematics during either semester of their junior year. In our first class, the students took the course for the entire semester. We now permit them to discontinue the course if they can pass the Stanford Test at the end of twelve weeks. Those unable to pass at the end of a semester are required to enroll for a second semester—exceptions are made in a few cases where schedule difficulties make this impossible. After two

semesters, whether a student passes or not his final score becomes a part of his permanent record.

Although a passing score in Refresher Mathematics is not required for graduation, most of our students are aware that if they cannot make a score of at least 72, they are deficient in mathematics. They realize too that to take their rightful place in society they need to know at least this much arithmetic. Their attitude towards the course indicates an awareness of the need for it.

We think we need Refresher Mathematics in our school. Too many of our graduates leave school with an insufficient knowledge of arithmetic. If we can make them realize that in the new era now approaching mathematics is the key to new discoveries and inventions we will have done our best, but even if we can only give them the arithmetic which they need in society it is still worth while.

In conclusion, our mathematical training must begin in the elementary schools. We must try there, more than ever before to teach every student to like arithmetic so that when he reaches high school he will want to take not just one or two, but three or even four years of algebra, geometry and other high school mathematics. When that happens, and not until that happens, I feel that we can discontinue Refresher Mathematics.

OUTDOOR NATURE COURSES FOR TEACHERS

Field Courses in Nature Study and Conservation will be conducted *at cost* by the National Audubon Society, at the AUDUBON NATURE CAMP near Medomak, Maine, and the AUDUBON NATURE CENTER of Greenwich, Connecticut during the summer of 1946.

The purpose of these courses is to help teachers understand the interrelationships between plants, animals, soil, and water through first hand observation of plant and animal life in a variety of natural habits; and by means of demonstrations and staff conferences, enable each teacher to develop an interesting and practical nature study and conservation program for use in his own school.

At the AUDUBON NATURE CAMP on a 330 acre wooded island in Muscongus Bay, Maine—teachers may enroll for one or more of five two-week sessions, June 14–August 29 (8th Season). Write for illustrated folder.

At the AUDUBON NATURE CENTER comprising 423 acres of wooded and brushy open land near Greenwich, Connecticut—teachers may enroll for one or more of four two-week sessions and one three-week session, June 10–Aug. 31 (3rd Season). Write for illustrated folder.

Address all inquiries to—SUMMER SESSIONS, National Audubon Society, 1006 Fifth Avenue, New York 28, N. Y.

MORAL VALUES AND PHYSICS

ORA WALTERS
Sewanhaka High School, Floral Park, L. I., N. Y.

It is unusual to associate moral values with science. In a machine age such as this, overt emphasis is given to skills, technical knowledge, electronics, radar, etc. Perhaps in our rush to produce—thus winning the war quickly—we have neglected some of the more permanent human values. Yet a student of science has some of these values suggested to him constantly.

For example the Law of Machines suggests a basic rule of life. In physics we say "Neglecting friction, work output equals work input." The same rule is suggested countless times—"You get out of an activity just what you put into it."

Then too, the scientific conception of efficiency has an application outside the physics laboratory. Witness for example the average student about to undertake some homework. He goes through a process of activity compared with which the efficiency of a steam engine is high. First he calls a friend to get the assignment (and of course forgets to ask for it), then he has to sharpen the pencil properly, next he has to adjust the light, look for paper etc.—a series of activity nine-tenths of which is wasted: a large input with a very small *useful* output.

Doesn't friction have its counterpart in human relationship? As with a machine, energy which is lost is dissipated in the form of heat. Just a little lubrication—or a little "soft soap" helps so much to produce a more productive relationship.

Turn to electricity and consider Ohm's Law. In part it states that a greater flow of current occurs where there is less resistance. Doesn't every teacher feel at times as if the students have erected a barrier against learning—and the greater the barrier the lesser the flow of information? A large share of the teacher's task is to overcome that barrier. Another part of the same law deals with the effect of voltage on current. Here is a lesson for us teachers; success in any project is directly proportional to the effort. An old adage carries out the same idea, "If at first you don't succeed. . . . "

Lenz's Law illustrates a fundamental tenet. "The induced current sets up a magnetic field which opposes the magnetic field causing the induced current." Isn't it so that any task offers reluctance, that it requires an effort, that "you can't get some-

thing for nothing?" It is not old fashioned to believe that one must strive to succeed.

A farmer was bringing in a load of freshly picked, sorted and packed apples. It was the last load of the season. It had been a bumper crop and the price was right. As he was rounding a slight incline with the wagon one wheel hit a stone, another hit a hole, over went the load and baskets of apples flew in all directions. The farmer righted the wagon and proceeded to salvage as many of the baskets of apples as were fit. "No great gain without some small loss" he philosophically muttered.

The same principle applies in science with respect to the electric motor and back emf. Here too there must be a loss if we are to gain. The gain in this case is simply the transformation of energy from one form to another.

Galileo, Newton and the others have contributed their scientific principles. Even a casual observer will find parallels in human relationships. Too, applications may be found in chemistry, biology and mathematics.

Essentially, the high school student is a moral, religious being. He attends church or Sunday school and he does so because of a deep seated interest in the basic principles of human conduct. It is at this time better than any other that he can be taught the philosopher's "long view." Some call it a proper sense of values; sometimes it is termed "common sense." At any rate it is the ability to look at a single incident and judge it on the basis of its effect on the pattern of life—rather than on the effect it may have on the pattern of a single day or a single mood. We hear the attitude expressed callously "A hundred years from now it won't make any difference." Scientists call it the "scientific attitude"—the habit of evaluation and scrutiny, the technique of putting each bit of data in its proper place and giving it proper weight. It is the ability to grope beneath the maze of information and grasp the fundamental issue.

Physics concerns itself with this attitude. The student of physics sees the reduction to fundamentals—that the life of man has been concerned with the development of devices, tools and so-called compound machines. It is evident that though these machines become as complicated as a B-29, yet they operate by the same fundamental law as the Model T. In like manner, complex as our living may become, fundamental values do not change. A course in physics can help to acquaint students with these values.

O. D. FRANK VISITS FLORIDA

[The following is a letter from our good friend and past-president, O. D. Frank, the apple man. When president he wrote "Post cards" for the Journal. The past winter he spent in Florida, and is so enthusiastic about the Southland that he bribed the Ed with the hint that he might send a coconut and a bit of fruit if the letter could be published. Now Ye Ed can't get the home folk of the South to tell about themselves and their surroundings—it is all so common-place! The letter follows. Is it old stuff to you north of Mason and Dixon's line? O. D. hopes our readers in the golden West, and also farther north in the apple country will just forget this. But after the letter was all set up in type along came the express man with a great big basket of tree-ripened fruit, some of Florida's best. That's why the Journal is late this month, but look out Florida, Here I Come. G. W. W.]

Dear Glen:

This is the first cloudy and rainy day we have had since coming to Florida, January 11. I am all eyes and ears in this new land that abounds with sunshine, flowers, fruits, birds and friendly folks. Many of the trees and other plants are new and strange. Others I have seen in green houses only. Blossoms nod at me with merry twinkles as if to say, "Bet you don't even know the family to which we belong." My reply is "you are right—got me—you might belong to the Jones or Warner family for all I *don't* know." I scratch my head, leaf through Gray—then ask a native.

Fruits fascinate me always, and while I still think of the apple as being the king of fruits and the great commoner among fruits, these citrus fellows are real likable chaps. When one picks and eats a tree ripened citrus fruit his palate is ruined for "color added" oranges. The oranges are picked green to insure safe shipping—color is added by a heating process and not by an "Easter dye" coloring material as I had thought.

The birds here are my pride and joy—I can identify some of them, warblers, towhees, grackles, sparrows, including our so-called English sparrow, who is a permanent resident in Florida—mocking birds that sing both day and night. Of course it may not be the same bird. The bird most in evidence everywhere is our dear old robin. Countless hundreds move from place to place in great flocks reminding us of the gangs of blackbirds in our corn fields in autumn.

Robins do not sing while they are here and of course they build no nests. They spend their time eating and playing. They are especially fond of the red berries of the Brazilian pepper tree which grows everywhere in Florida. I have seen more robins here in a day than I have in fifty years in Indiana and Illinois.

Among the numerous water birds, the pelicans and cormorants have attracted us most. It seems to me that about every second picture post card we send back home has the picture of a pelican with that world known rhyme on it. Wonder who in the world the author is. Gulls, terns and ducks may be seen along the coast anywhere—clean, graceful, *hungry* birds.

Last week I had the privilege of taking groups of school children on field trips. We used the "Big Five Field Trip" technique. When we visited an orange grove, i.e., eyes, ears, nose, touch and taste—the five big avenues nature has provided for our education. An orange tree has so much more to tell us *out of a look* than out of a book—education on the hoof if you please.

The children were enthusiastic about the field trips. Oranges never had tasted so good, looked so good, felt so good, and orange blossoms were never so fragrant. "Can't we go on other field trips" was the wish expressed by each group.

I am sorry to learn that little time is spent out of doors with the children even in this "Flowery Day" land of sunshine where so much is offered in the very doorway of every school house.

In the dozen or more schools I have visited scarcely any field work is done in the biology or nature-study classes, and I fear that this is true of schools more or less the country over. Were I teaching biology here in the southland the out of doors would be my classroom and laboratory.

Near the town of Palmetto in Manatee County, which is one of the most productive sections of Florida, one of my former Quincy, Illinois High School students, Margaret Turner Menick and her husband Wayne, own twenty acres of the finest garden land in the country.

Margaret's interest in the out of doors with birds as her hobby, prompted her to suggest that we lay out a nature trail—a "Trail Site Museum" we call it, for the school children of Bradenton and Palmetto.

I must confess that there were a number of plants that neither Margaret nor I knew. Fortunately a fine old colored grandmother who has spent her life near Palmetto happened along and assisted us by giving the common names of a number of the plants.

You should have heard her chuckle when she "educated" us. Really, I havent seen an unhappy Negro since I have been in Florida. Their perennial good cheer is a joy to see.

The trail passes along the edge of an orange grove just before it enters Wayne and Margaret's woods, composed of live oaks festooned and bewhiskered with Spanish moss, palmettos, hock berries, cat brier, ferns and other tangled plants.

Mr. Phillips, who owns the orange grove has generously given us permission to place an orange from his trees in the hands of each boy and girl whom we take through the "Museum" thus leaving a good taste in all of our mouths.

In a swamp near the orange grove a great old alligator gave loud grunts as we passed by. My hair—few—stood a little straighter but Wayne assured me that 'gators stay near the water out of sight. Some of the frogs in the swamp, too, have voices that boom out sizeable "Jug-a-rums."

Last week Mrs. Frank and I visited my cousin May Frank Rhoads and her good husband Joe who have a three acre "ranch" near Lake Worth five miles south of Palm Beach.

May is the only Frank I know of who owns coconut trees. If our ancestors, who were "educated in the higher branches," could have seen me gathering nuts from the great clusters in May's coconut trees, I am sure they would have recognized at once that I run true to form. Removing the outer hulls or husks from coconuts is about the most difficult job I have tackled since I came to Florida.

I must tell you about the "bug" situation in Florida. We came with fear and trembling. Mrs. Frank with the fear that there would be too many—certain newspapers had warned us—and I feared that I might not find many new varieties. Both fears were unfounded. There are not many insects here that bother people *personally*, but a host that make farming and gardening a real problem. Ants and aphids go hand in hand, or should I say, mandible in mandible, to get the juice—sap—from corn, cabbage, lettuce, beans, melons and other field and garden crops. No orange, grapefruit or lemon grove is free of these pesky pests. Everywhere they multiply by countless thousands, *all females*, born alive on the leaves of trees and other plants. The aphids suck the sap from the plants and the ants lap up the "honey dew" excreted by the aphids. It is difficult to poison these aphids or plant lice because their bills (probosces) enable the insects to feed from the interior of the leaf.

We have not been pierced by a single mosquito and we have seen less than a dozen house flies. I have found only two roaches and I had to hunt for them. There are plenty of ants but none have bitten me although I have tramped around the country up to my neck in the "jungles." The country is alive with doodle bug (ant lion) homes. These fellows deserve a medal for helping to keep the ant population in check.

Butterflies are plentiful; except for cabbage butterflies and two monarchs, evidently migrants, the whole crowd is new to me. Were it possible to call back some years I would be out there with my butterfly net jumping all over the place making a collection. Seeing all of these beauties arouses the collecting "instinct" like unto the old retired fire horse that responds to a fire call even when he is hitched to a milk wagon.

Sincerely yours,

O. D. FRANK

FORTY-ONE MILLION SPENT FOR RESEARCH

During 1945 the Westinghouse Electric Corporation utilized $41,645,819 for scientific and engineering research. Of this amount the Company spent $2,065,395 for "pure" research—experiments dedicated to increasing scientific learning—and the remainder was provided by Westinghouse and its customers to bring products developed by research to the production stage.

If he had nothing to say, what was the good of saying it in three languages?—Dodsworth.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON

State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

3. In general when several solutions are correct, the ones submitted in the best form will be used.

LATE SOLUTIONS

1957. Edgar H. Rose, Rochester, N. Y.; Herbert Reisman, Chicago.

1958. Walter R. Warne, Columbia, Mo.

1962. Louise Wilson, Edmonton, Canada; Emily Warne, Romulus, N. Y.; Alice Lisk, Bennington, Va.; Hazel S. Wilson, Annapolis, Md.

1963. *Proposed by Grace Marsh, Mexico City, D. F.*

If $\cos 2y = \tan(45 - x)$, show that $\tan x = \tan^2 y$.

Solution by Joseph Lerner, New York City

$$\cos 2y = \tan(45 - x)$$

$$\cos 2y = \frac{1 - \tan x}{1 + \tan x}$$

solving for $\tan x$:

$$\tan x = \frac{1 - \cos 2y}{1 + \cos 2y}$$

$$= \frac{1 - \cos^2 y + \sin^2 y}{1 + \cos^2 y - \sin^2 y}$$

$$= \frac{2 \sin^2 y}{2 \cos^2 y}$$

$$= \tan^2 y.$$

Solutions were also offered by Francis L. Miksa, Aurora, Ill.; Hazel S. Wilson, Annapolis, Md.; Wm. A. Richards, Berwyn, Ill.; M. Kirk, Media,

Pa.; Helen M. Scott, Baltimore, Md.; Felix John, Philadelphia, Pa.; Rose U. Brate, Canandaigua, N. Y.; Br. Philip, Montreal, Canada; Mrs. Walter R. Warne, Columbia, Mo.; T. Theodore, St. Louis, Mo.; V. C. Bailey, Delaware, Ohio; U. Alfred, Napa, Calif.

1964. *Proposed by Lillian A. MacDonald, Newark, N. J.*

If S_p is the sum of Pth powers of the first n consecutive integers, then $9(S_8-S_4)$ divided by $5(S_4-S_2)$ equals the square of a whole number.

Solution by Francis T. Miksa Aurora, Ill.

From Smithsonian Mathematic Formulae, we get from formula #1.875 p. 27

$$S_8=\frac{n^9}{9}+\frac{n^8}{2}+\frac{2n^7}{3}+0-\frac{7n^5}{15}+0+\frac{2n^3}{9}+0-\frac{n}{30}$$

$$S_4=\qquad\qquad+\frac{3n^5}{15}+\frac{n^4}{2}+\frac{n^3}{3}\qquad-\frac{n}{30}$$

$$S_2=\qquad\qquad\qquad\qquad+\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$$

Whence

$$9(S_8-S_4)=\frac{2n^9+9n^8+12n^7-12n^5-9n^4-2n^3}{2}$$

$$5(S_4-S_2)=\frac{2n^5+5n^4-5n^2-2n}{2}$$

Set

$$M=\frac{9(S_8-S_4)}{5(S_4-S_2)}=\frac{n^2(2n^6+9n^5+12n^4-12n^2-9n-2)}{(2n^4+5n^3-5n-2)}$$

$$M=\frac{n^2(n^2-1)(n+1)(2n^3+7n^2+7n+2)}{(n-1)(2n^3+7n^2+7n+2)}$$

$M=n^2(n+1)^2=$ a square of a whole number.

Solutions were also offered by Helen M. Scott, Baltimore, Md.; Felix John, Philadelphia, Pa.; W. Nicholson, Chicago; Br. Philip, Montreal, Canada; M. M. Dreiling, Collegeville, Ind.

1965. *Proposed by Brother Felix John, Philadelphia, Pa.*

Solve the system

$$x-y+z+w=\ 8 \tag{1}$$

$$x^2+y^2-z^2-w^2=36 \tag{2}$$

$$xy+zw=42 \tag{3}$$

$$x^3-y^3-z^3-w^3=\ 0 \tag{4}$$

Solution by U. Alfred, Napa, Calif.

Solution. Subtract twice (3) from (2) and put the result in the form:

$$(x-y)^2-(z+w)^2=-48$$

or

$$(x-y+z+w)(x-y-z-w)=-48$$

Since

$$x-y+z+w=8 \quad \text{by (1)},$$

then

$$x-y-z-w=-6$$

Solving for $x-y$ and $z-w$, we obtain

$$x-y=1$$
$$z+w=7$$
$$(x-y)^3-(z-w)^3=x^3-3x^2y+3xy^2-y^3-z^3-3z^2w-3zw^2-w^3=1-343=-342.$$

Using (4), this becomes:

$$-3xy(x-y)-3zw(z+w)=-342.$$

Substituting for $x-y$ and $z+w$,

$$xy+7zw=114$$

By (3)

$$xy+\ zw=\ 42.$$

whence

$$xy=30, \qquad zw=12.$$

Substituting for w

$$7z-z^2=12 \quad \text{from which} \quad z=3,\ 4$$
$$w=4,\ 3$$

$$y^2+y-30=0$$

Similarly substituting for x

$$y=-6,\ 5 \qquad x=-5,\ 6$$

Checking values in (4), the following table of solutions is obtained:

x	y	z	w
-5	-6	3	4
-5	-6	4	3
6	5	3	4
6	5	4	3

Solutions were also offered by V. C. Bailey, Delaware, Ohio; Walter R. Warne, Columbia, Mo.; Helen M. Scott, Baltimore Md.; M. Kirk, Media, Pa.; Wm. A. Richards, Berwyn, Ill.; Hazel S. Wilson, Annapolis, Md.; Nicholas Lagotheto, R. I.; Br. Philip, Montreal, Canada; Francis L. Miksa, Aurora Ill.; Mrs. Walter R. Warne, Columbia, Mo.; and the proposer.

1966. *Proposed by Isaac Conley, Richmond, Va.*

The base of a triangular pyramid, $V-ABC$, is an equilateral triangle. If the faces are inclined at an angle of θ to the base, show that the angle between the lateral edges is $2 \arctan(\sqrt{3}\cos\theta)$.

Solution by V. C. Bailey, Delaware, Ohio

Let

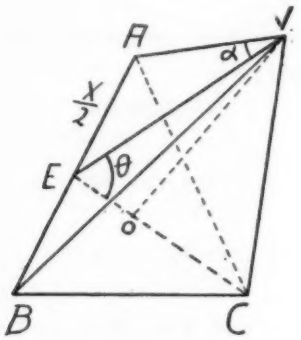
(1) x=side of the base.

then

(2) $$\frac{x}{6}\sqrt{3}=EO$$

(3) $$VE=\frac{EO}{\cos\theta}=\frac{x\sqrt{3}}{6\cos\theta}$$

O is the median point of the base. 2α is the desired angle.



(4) $$\tan\alpha=\frac{\frac{x}{2}}{VE}=\sqrt{3}\cos\theta$$

(5) $$\alpha=\arctan(\sqrt{3}\cos\theta)$$

(6) $$2\alpha=2\arctan(\sqrt{3}\cos\theta).$$

Solutions were also offered by Wm. A. Richards, Berwyn, Ill.; Felix John, Philadelphia, Pa.; Helen M. Scott, Baltimore, Md.; Francis L. Miska, Aurora, Ill.; Br. Philip, Montreal, Canada; U. Alfred, Napa, Calif.

1967. *Proposed by Nellie Bishop, Seneca Falls, N. Y.*

In triangle ABC, if P is a point such that $BP=a/4$, show that tan $BAP=\sqrt{3}/7$.

The position of P is not given specifically, and hence there is no solution. The problem was taken from Hobson and Jessop's Elementary plane trigonometry.

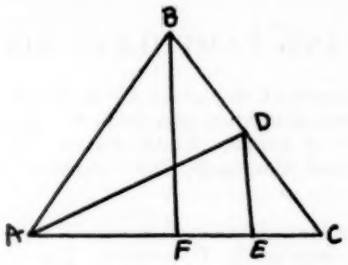
1968. *Proposed by Blanche Yerkes, Bearytown, New York.*

If $AB=BC$ in triangle ABC and if D is the mid point of BC, show, without use of tables that

$$\cot DAC=3.$$

Comments by D. F. Wallace, St. Paul, Minn.

Since DE and BF are parallel, being drawn perpendicular to AC, and since D is mid point of BC, DE bisects FC. Also BF bisects AC. Then $FE=EC=AE/3$. Now if cot $DAC=3$, $DE=AE/3$ and $DE=EC$. This makes DEC a right triangle with $\angle C=45°=\angle CBF$. Thus $\angle B=90°$. The problem is thus seen to be true only if $\angle B=90°$.



Similar reports were made by Helen M. Scott, Baltimore, Md.; M. Kirk, Media, Pa.; Felix John, Pniladelphia, Pa.

HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

1959, 62. B. J. Kirby, Upper Canada College, Toronto

1957. H. F. Trotter, Upper Canada College, Toronto

1961. J. P. Mayberry, Upper Canada College, Toronto

1963. D. A. Barr, Upper Canada College, Toronto

PROBLEMS FOR SOLUTION

1981. *Proposed by J. F. Arena, Boone, N. C.*

Evaluate $\int x^3\sqrt{x+a}\,dx$ without use of table of integrals.

$$\cot DAC = 3.$$

1982. *Proposed by Milton Schiffenbauer, Camp Wolters, Texas.*

The sum of the squares of 13 consecutive integers cannot be a perfect square.

1983. *Proposed by Howard D. Grossman, New York City.*

Find the sum to infinity of the series $1/2+1/3-1/4-1/5+1/6+1/7$

1984. *Proposed by E. N. Shuster, Trenton, N. J.*

From a point within a square three lines are drawn, one to a vertex. If the lengths of the lines are 300, 400 and 500, find the length of a side of the square.

1985. *Proposed by Walter R. Warne, Columbia, Mo.*

Solve the system:

$$\frac{x}{y^2-3}=\frac{y}{x^2-3}=\frac{-7}{x^3+y^3}.$$

1986. *Proposed by Agnes Dunlap, Providence, R. I.*

If the diagonals of a cyclic quadrilateral are perpendicular to each other, the line through their point of intersection perpendicular to one side bisects the other side.

BOOKS AND PAMPHLETS RECEIVED

ESSENTIALS OF GENERAL CHEMISTRY, by B. Smith Hopkins, *Professor of Inorganic Chemistry, Emeritus*, and John C. Bailer, Jr., *Professor of Chemistry, University of Illinois*. Cloth. Pages v+520. 14.5×22.5 cm. 1946. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $3.50.

THE PRINCIPLES OF HEREDITY, by Laurence H. Snyder, Sc.D., *Professor of Zoology and Chairman of the Dpeartment, The Ohio State University*. Third Edition. Cloth. Pages xvi+450. 14.5×22.5 cm. 1946. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $3.75.

ELEMENTS OF CALCULUS, by William Anthony Granville, Ph.D., LL.D. *Formerly President of Gettysburg College*, Percey F. Smith, Ph.D., and William Raymond Longley, Ph.D., *Professors of Mathematics, Yale University*. Cloth. Pages xi+549. 15×23 cm. 1946. Ginn and Company, Statler Building, Boston, Mass. Price $3.75.

PLANE GEOMETRY, by Rachel P. Keniston and Jean Tully, *Stockton High School, Stockton, California*. Cloth. Pages vii+392. 17.5×24 cm. 1946. Ginn and Company, Statler Building, Boston, Mass. Price $1.88.

ANALYTICAL GEOMETRY, by Francis D. Murnaghan, Ph.D. (Johns Hopkins), D.Sc. (Nat. Univ. Ireland), *Professor of Applied Mathematics, The Johns Hopkins University*. Cloth. Pages viii+402. 15×23 cm. 1946. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price $3.25.

SECONDARY EDUCATION FOR VETERANS OF WORLD WAR II, by Gene Oppy, *Howe Military School;* John Ramseyer, *University School;* Harold Reynard, *University School;* Margaret Willis, *University School Editor*. Paper. 112 pages. 15×23 cm. 1945. The Ohio State University, Columbus 10, Ohio. Price 50 cents.

RCA TELEVISION POLICIES AND PLANS. Statements by Brigadier General David Sarnoff, *President, Radio Corporation of America;* Frank M. Folsom, *Executive Vice President in Charge of RCA Victor Division;* Niles Trammell, *President, National Broadcasting Company;* and Dr. C. B. Jolliffe, *Executive Vice President in Charge of RCA Laboratories*. Paper. 24 pages. 14.5×21 cm. 1945. Radio Corporation of America, 30 Rockefeller Plaza, New York, N. Y.

BOOK REVIEWS

ESSENTIAL VOCATIONAL MATHEMATICS, By Claude H. Ewing and Walter W. Hart, pages 1–266. 1945. D. C. Heath & Company, Chicago, Illinois, Price $1.60.

In the preface of this book the author states that the book is prepared for a first year course for vocational students. It is well illustrated by drawings and instruments used in a shop course. This book contains a good review of the fundamentals of arithmetic, and by the use of specific problems from shop it adds some practical geometry, algebra, and trigonometry in such a way that the student at least sees the uses of these topics in the field of shop.

The table of content lists the following eight topics: Integers and Deci-

mals, Common Fractions, Computing With Measures, Percentage, Practical Geometry, Perimeter-Area-Volume, Practical Algebra, and Practical Trigonometry. Also a supplement on the Metric System of Measures and four pages of tables—squares, square roots, cubes, cube roots, measures of length, area, volume, capacity, weight, angles, and arcs, and values of the trigonometric ratios—sine, cosine, tangent, cotangent. A good many exercises are given with each topic presented.

In a school where the majority of the General Mathematics pupils are taking shop this book would be a valuable text. For other mathematics classes it would be a good reference.

C. M. BENNETT
Concannon High School,
West Terre Haute, Ind.

GAINING SKILL IN ARITHMETIC, by Benjamin Braverman, *Chairman of the Department of Mathematics, Seward Park High School, New York City*, pages 1–134. 1945. D. C. Heath and Company $1.40. Pupils' graph & mastery Tests by same author, pages 1–80.

In the foreword to the teachers the author states that this book is for those pupils who come to high school but who have not attained seventh grade arithmetic achievement. This book is so written that it may be used as a basic text in remedial arithmetic classes where each pupil may be permitted to work at his own rate of speed. Each unit of work has explanations and illustrative examples followed by three sets of 10 practice examples. The first set of 10 is to be worked, checked by the answers in the text, then followed by the mastery test Form A on that unit. If the quality of work on this test is unsatisfactory sets 2 and 3 are to be worked from the text and mastery test Form B given to the pupil.

The book contains the following thirteen units of work: Addition and Subtraction of Whole Numbers, Multiplication of Whole Numbers, Division of Whole Numbers, Common Fractions, Meaning, Reduction, Addition and Subtraction of Common Fractions, Meaning of a Decimal Fraction, Computing with Decimals, Meaning of Per cent, Finding a Given Per cent of a Number, Finding What Per cent One Number is of Another, Computing With Measures, and Meaning and Use of Ratio.

This book would be a valuable text for a teacher who needed plenty of exercises on a given topic and several tests already made out on those topics. It would make a good supplementary text for pupils who need to improve and to keep that level of improvement in arithmetic.

C. M. BENNETT

ELECTRONICS FOR ENGINEERS, Edited by John Markus and Vin Zeluff, *Associate Editors, Electronics*. First Edition. Cloth. Pages x+390. 21.5×28 cm. 1945. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York 18, N. Y. Price $6.00.

In this book the Associate Editors of "Electronics" magazine have done the electronics engineers a great service. They have here made a collection of 142 articles, with complete charts and graphs, that have appeared in "Electronics" since its beginning in 1930. The majority of the articles are ones that have appeared within the past five years.

An engineer specializing in a certain field usually accumulates considerable "know-how" and design methods which can often be summarized in charts and graphs. Many engineers are willing to share their stock-in-trade with others through publication. "Electronics" has served as one of the major means of exchange of knowledge in the electronics field. These

design articles serve as a short cut for new engineers entering the field. It becomes very laborious, however, for each engineer to hunt through many bound volumes of periodicals for the various articles in his field. In this book the editors have collected many of the most useful articles and grouped them into twenty-seven different electronic design fields. The authorship of each article is given and the date of the original publication is given in the contents pages.

This is not a textbook covering the general field of electronics, nor does each article attempt to give the general theory of its topic, but rather deals with specific design problems with special methods for solution. The book, therefore is in the nature of a handbook that should be on every electronics design engineer's desk for ready reference.

Many may not like the new double sized pages that seem to be creeping into textbooks, but in this case seems to be justified to give space for graphs and charts of sufficient size to be easily read. The printing is in three column form for ease of eye-span.

H. R. VOORHEES
Chicago Junior College,
Herzl Branch

APPLIED MATHEMATICS FOR RADIO AND COMMUNICATION ENGINEERS, by Carl E. Smith, *President, Smith Practical Radio Institute, Cleveland.* Cloth. Pages x+336. 13×21 cm. 1945. McGraw-Hill Book Company, Inc., New York 18, N. Y. Price $3.50.

This is the first in a series of text books on radio and communication engineering. The author states that these volumes may serve as a refresher course for persons already familiar with the subject or as a study guide for those desiring a comprehensive treatment of the material. The course is intended for the student who has the equivalent of a high school education and is familiar with the popular terminology used in radio.

The text presents a clear and simple development of the principles of mathematics used in radio and communication engineering beginning with arithmetic and following up through an excellent treatment of calculus. The latter topic occupies three chapters of the book. The first of these deals with differential calculus in elementary and graphical forms which treat algebraic functions; the second deals in a similar way with transcendental functions; while the third presents a treatment of elementary and graphical forms of integral calculus. Other chapters include discussions of such topics as logarithms, vector addition, complex quantities, curves and graphs, simultaneous equations, quadratic equations and hyperbolic trigonometry. The book concludes with a chapter on series and wave forms which is intended to meet the modern trends to frequency modulation, television and radar development.

The material is so arranged that the fundamentals may be mastered easily by self-study. A number of well chosen practice examples following the inductive development of each theory make it possible for the student to master the practical application of the principles studied. Some 95 excellent illustrations aid greatly in the development of a clear understanding of the various topics presented. The text is made to be self-contained by the inclusion of 84 pages of tables, formulas, abbreviations and symbols, a number of useful constants, and wave forms. It is the intention of the author to refer to this material in other books of the series whenever the need arises.

JAMES B. MAUS
Lyons Twp. High School
La Grange, Illinois